

# **MATHEMATICS CURRICULUM FRAMEWORK**

**Connecticut State Department of Education  
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# MATHEMATICS

*By the end of Grade 12, students will apply proficiently a range of numerical, algebraic, geometric and statistical concepts and skills to formulate, analyze and solve real-world problems; to facilitate inquiry and the exploration of real-world phenomena; and to support continued development and appreciation of mathematics as a discipline.*

## PROGRAM GOALS

As a result of education in Grades K-12, students will:

- communicate numerical, geometric, algebraic and statistical ideas orally and in written form with models, pictures, graphs and mathematical symbols, using paper and pencil, a variety of calculator displays, spreadsheets, graphing packages, word processing and other related computer software;
- use inductive and deductive reasoning to make, defend and evaluate conjectures and arguments, to justify assertions and verify tentative conclusions, and to solve mathematical problems;
- use mathematical skills and concepts to make and justify decisions and predictions, to identify patterns and trends, to pose questions from data and situations, and to formulate and solve problems;
- identify and use connections within mathematics to identify interrelationships and equivalent representations, to construct mathematical models, and to investigate and appreciate mathematical structure;
- use mathematical skills and concepts to describe and analyze data and measurements from other disciplines;
- select and use appropriate approaches and tools for solving computational, geometric and algebraic problems, including estimation, mental computation, guess and test, paper and pencil, calculators and computers with software for tabulating, charting, graphing, drawing and transforming data and images; and
- use mathematical skills and concepts with proficiency and confidence, and appreciate the power and utility of mathematics as a discipline and as a tool for solving problems.

# K-12 CONTENT STANDARDS

- 1. Number Sense**

Students will use numbers to count, measure, compare, order, scale, locate and label, and use a variety of numerical representations to present, interpret, communicate and connect various kinds of numerical information.
- 2. Operations**

Students will add, subtract, multiply and divide with whole numbers, fractions, decimals and integers, and develop strategies for selecting the appropriate computational and operational methods for solving problems.
- 3. Estimation and Approximation**

Students will make estimates and approximations, and judge the reasonableness of results.
- 4. Ratios, Proportions and Percents**

Students will use ratios, proportions and percents to represent relationships between quantities and measures and solve problems involving ratios, proportions and percents.
- 5. Measurement**

Students will make and use measurements in both customary and metric units to approximate, measure and compute length, area, volume, mass, temperature, angle and time.
- 6. Spatial Relationships and Geometry**

Students will analyze and use spatial relationships and basic concepts of geometry to construct, draw, describe and compare geometric models and their transformations, and use geometric relationships and patterns to solve problems.
- 7. Probability and Statistics**

Students will use basic concepts of probability and statistics to collect, organize, display and analyze data, simulate events and test hypotheses.
- 8. Patterns**

Students will discover, analyze, describe, extend and create patterns, and use patterns to describe mathematical and other real-world phenomena.
- 9. Algebra and Functions**

Students will use algebraic skills and concepts, including functions, to describe real-world phenomena symbolically and graphically, and to model quantitative change.
- 10. Discrete Mathematics**

Students will use the concepts and processes of discrete mathematics to analyze and model a variety of real-world situations that involve recurring relationships, sequences, networks, combinations and permutations.

**CONTENT STANDARD 1: Number Sense**

*Students will use numbers to count, measure, compare, order, scale, locate and label, and use a variety of numerical representations to present, interpret, communicate and connect various kinds of numerical information.*

**K-12 PERFORMANCE STANDARDS**

Educational experiences in **Grades K-4** will assure that students:

- use real-life experiences, physical materials and technology to construct meanings for whole numbers, commonly used fractions and decimals;
- understand our numeration system by modeling, counting, grouping and using place-value concepts;
- use numbers to count, as measures, labels and as indicators of location;
- use models and pictures to demonstrate understanding of equivalent forms of numbers;
- understand and use properties of numbers, including odd, even, ordinal and cardinal; and
- develop a sense of magnitude of numbers by ordering and comparing whole numbers, commonly used fractions, decimals and money amounts.

Educational experiences in **Grades 5-8** will assure that students:

- use real-life experiences, physical materials and technology to construct meanings for whole numbers, commonly used fractions, decimals and money amounts, and extend these understandings to construct meanings for integers, rational numbers, percents, exponents, roots, absolute value and scientific notation;
- model, represent and use numbers in a variety of equivalent forms (integer, fraction, decimal, percent, exponential and scientific notation) as they arise from real-world situations;
- use the equivalence of fractions, decimals and percents to select appropriate and efficient ways to write, order, compare, estimate and compute;

Educational experiences in **Grades 9-12** will assure that students:

- use real-life experiences, physical materials and technology to construct meanings for rational and irrational numbers, including integers, percents and roots;
- use number sense and the properties of various subsets of real numbers to solve real-world problems;
- develop and use an intuitive sense of the magnitude of numbers (including very large and very small numbers) and relate them to place value and exponential forms; and
- select an appropriate form to represent and use numerical data (integer, fraction, decimal, ratio, percent, exponential, scientific notation, irrational, complex) as they arise from real-world situations involving magnitude, order, measures, labels, locations and scales.

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**K-12 PERFORMANCE STANDARDS, continued**

Educational experiences in Grades 5-8 will assure that students:

- develop and use a sense of order and magnitude of fractions, decimals, integers, powers and roots; and
- develop and apply number theory concepts (primes, factors, multiples and divisibility rules), as appropriate, in various real-world problem situations.

## CONTENT STANDARD 2: Operations

*Students will add, subtract, multiply and divide with whole numbers, fractions, decimals and integers and develop strategies for selecting the appropriate computational and operational methods for solving problems.*

### K-12 PERFORMANCE STANDARDS

Educational experiences in Grades K-4 will assure that students:

- develop meaning for the operations by modeling, comparing and discussing a variety of problem situations;
- develop proficiency with basic addition, subtraction, multiplication and division facts through the use of a variety of strategies and contexts;
- use informal language, mathematical language and symbols to relate problem situations to operations;
- recognize that any one operation can be used to represent diverse problem situations, e.g., subtraction can be used in "take away," as well as comparison, situations;
- construct, use and explain a variety of procedures for performing whole number calculations; and
- understand and use relationships among operations, e.g., multiplication is repetitive addition; multiplication is the opposite of division.

Educational experiences in Grades 5-8 will assure that students:

- maintain proficiency with basic addition, subtraction, multiplication and division facts through the use of a variety of strategies and contexts;
- develop, use and explain procedures for performing calculations with whole numbers, decimals, fractions and integers;
- understand the concepts of powers and roots, and apply them in problem situations;
- select and use an appropriate method for computing from among mental math, estimation, paper-and-pencil and calculator methods; and
- use relationships among operations and properties of operations (associative, commutative and distributive) as well as order of operations and inverses to simplify computations.

Educational experiences in Grades 9-12 will assure that students:

- use arithmetic operations to solve problems encountered in everyday consumer situations;
- apply and explain procedures for performing calculations with whole numbers, decimals, fractions and integers;
- use appropriate methods for computing, including mental math, estimation, paper-and-pencil and calculator methods;
- use field properties and the relationship between operations and their inverses to justify mathematical procedures; and
- use absolute value, powers and roots; explore and use negative exponents on integers.

## CONTENT STANDARD 3: Estimation and Approximation

*Students will make estimates and approximations,  
and judge the reasonableness of results.*

### K-12 PERFORMANCE STANDARDS

Educational experiences in Grades K-4 will assure that students:

- explore, construct and use a variety of estimation strategies;
- recognize when estimation is appropriate and understand the usefulness of an estimate as distinct from an exact answer;
- use estimation to determine the reasonableness of an answer;
- visually estimate length, area, volume and angle using various referents; and
- apply estimation when working with quantities, measures and problems.

Educational experiences in Grades 5-8 will assure that students:

- develop, apply and explain a variety of estimation strategies in problem situations involving quantities and measures;
- use estimation to predict outcomes and determine reasonableness of results;
- recognize when estimation is appropriate and understand the usefulness of an estimate as distinct from an exact answer; and
- determine whether a given estimate is an overestimate or underestimate.

Educational experiences in Grades 9-12 will assure that students:

- assess the reasonableness of answers to problems arrived at using pencil-and-paper techniques, mental math, formulas, calculators or computers;
- develop, use and apply a variety of estimation strategies in problem situations;
- make reasonable estimates of the values of formulas, functions and roots; and
- recognize the limitations of estimation and assess the amount of error resulting from estimation.

## CONTENT STANDARD 4: Ratios, Proportions and Percents

*Students will use ratios, proportions and percents to represent relationships between quantities and measures and solve problems involving ratios, proportions and percents.*

### K-12 PERFORMANCE STANDARDS

Educational experiences in Grades K-4 will assure that students:

- describe simple ratios when comparing quantities.

Educational experiences in Grades 5-8 will assure that students:

- understand and use ratios, proportions and percents in a wide variety of situations;
- develop, apply and explain methods for solving problems involving proportions and percents;
- use and differentiate between fractional parts and ratios when comparing quantities; and
- use dimensional analysis to identify and find equivalent rates.

Educational experiences in Grades 9-12 will assure that students:

- understand and explain the need for proportions and percents;
- use ratios, proportions and percents to solve real-world problems;
- use dimensional analysis and equivalent rates to solve problems;
- describe direct and indirect variation and apply them to numerical, geometric and algebraic models and related problems; and
- describe trigonometric ratios and apply them to measuring triangles.

## CONTENT STANDARD 5: Measurement

*Students will make and use measurements in both customary and metric units to approximate, measure and compute length, area, volume, mass, temperature, angle and time.*

### K-12 PERFORMANCE STANDARDS

Educational experiences in Grades K-4 will assure that students:

- use and describe measures of length, distance, capacity, mass, area, volume, time, temperature and angle;
- compare and order objects according to some measurable attribute;
- develop and use personal referents, such as fingers and arm spans, as estimates for standard units of measure; and
- select and use appropriate standard and nonstandard units of measurement to solve problems.

Educational experiences in Grades 5-8 will assure that students:

- estimate, make and use measurements to describe and compare phenomena, and explore the structure and use of systems of measurement, including converting units within systems;
- select and use appropriate measurement units and tools to make measurements to the degree of accuracy required by the situation;
- solve problems involving the concept of, calculation of, and relationships among length, perimeter, area, volume, angle measure, capacity, weight, mass and temperature; and
- develop and use formulas and procedures for solving measurement problems.

Educational experiences in Grades 9-12 will assure that students:

- extend, apply and formalize understandings of measurement, including strategies for determining perimeters, areas and volumes, and the dimensionality relationships among them;
- describe and apply the effect of a change in length on the area and volume of an object;
- choose appropriate tools and techniques to measure quantities to specified degrees of precision and accuracy;
- use techniques of algebra, geometry and trigonometry to measure quantities indirectly; and
- use and create scales and calibrations to solve problems involving measurement.

## CONTENT STANDARD 6: Spatial Relationships and Geometry

*Students will analyze and use spatial relationships and basic concepts of geometry to construct, draw, describe and compare geometric models and their transformations, and use geometric relationships and patterns to solve problems.*

### K-12 PERFORMANCE STANDARDS

Educational experiences in Grades K-4 will assure that students:

- describe, model, draw and classify shapes;
- investigate and predict the results of combining, subdividing and changing shapes;
- identify and use geometric shapes in various orientations, including rotations, reflections and translations;
- use real-life experiences, concrete objects and technology to explore and understand properties of 2- and 3-dimensional geometric shapes; and
- explore relationships among and properties of shapes, such as congruence, similarity and symmetry.

Educational experiences in Grades 5-8 will assure that students:

- investigate, explore and describe the geometry in nature and real-world applications;
- identify, visualize, model, describe and compare properties of and relationships among 2- and 3-dimensional shapes;
- describe and use fundamental concepts and properties of, and relationships among, points, lines, planes, angles and shapes, including incidence, parallelism, perpendicularity, congruence, similarity and the Pythagorean theorem;
- construct, analyze and apply the effects of reflections, translations, rotations and dilations on various shapes;
- relate 2- and 3-dimensional geometry using shadows, perspectives, projections and maps; and
- solve real-world problems using geometric concepts.

Educational experiences in Grades 9-12 will assure that students:

- use transformations, coordinates and vectors and appropriate computer software to explore and develop an understanding of Euclidean geometry;
- deduce properties of, and relationships among, figures from given assumptions;
- develop an understanding of an axiomatic system through geometric investigations, making conjectures, formulating arguments and constructing proofs;
- understand and analyze the geometry of three-dimensional shapes and their cross-sections;
- solve real-world and mathematical problems using geometric models; and
- interpret algebraic equations and inequalities geometrically, and describe geometric objects algebraically.

## CONTENT STANDARD 7: Probability and Statistics

*Students will use basic concepts of probability and statistics to collect, organize, display and analyze data, simulate events and test hypotheses.*

### K-12 PERFORMANCE STANDARDS

Educational experiences in Grades K-4 will assure that students:

- pose questions, make predictions and solve problems that involve collecting, organizing and analyzing data;
- construct, read and interpret displays of data such as pictographs and bar and circle graphs;
- make inferences and formulate hypotheses based on data;
- generate and analyze data obtained from such chance devices as spinners and dice;
- develop intuition about the probability of various real-world events; and
- make predictions that are based on intuitive and experimental probabilities.

Educational experiences in Grades 5-8 will assure that students:

- make conjectures; design simulations and samplings; generate, collect, organize and analyze data; and represent the data in tables, charts, graphs and creative data displays;
- make inferences and formulate and evaluate hypotheses and conclusions based on data from tables, charts and graphs;
- describe the shape of the data using range, outliers, and measures of central tendency, including mean, median and mode;
- select and construct appropriate graphical representations and measures of central tendency for sets of data;
- determine the probability of simple and compound events;
- model probabilistic situations using both simulations and theoretical methods;

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Educational experiences in Grades 9-12 will assure that students:

- estimate probabilities, predict outcomes and test hypotheses using statistical techniques;
- design a sampling experiment, interpret the data, and recognize the role of sampling in statistical claims;
- use the law of large numbers to interpret data from a sample of a particular size;
- select appropriate measures of central tendency, dispersion and correlation;
- design and conduct a statistical experiment and interpret its results;
- draw conclusions from data and identify fallacious arguments or claims;
- use scatterplots and curve-fitting techniques to interpolate and predict from data;

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## CONTENT STANDARD 7: Probability and Statistics

### K-12 PERFORMANCE STANDARDS, continued

Educational experiences in Grades 5-8 will assure that students:

- make predictions that are based on experimental and theoretical probabilities; and
- draw conclusions from data and identify fallacious arguments or claims.

Educational experiences in Grades 9-12 will assure that students:

- use relative frequency and probability to represent and solve problems involving uncertainty; and
- use simulations to estimate probabilities.

**CONTENT STANDARD 8: Patterns**

*Students will discover, analyze, describe, extend and create patterns and use patterns to describe mathematical and other real-world phenomena.*

**K-12 PERFORMANCE STANDARDS**

Educational experiences in Grades K-4 will assure that students:

- reproduce, extend, describe and create patterns and sequences using a variety of materials and attributes;
- use tables and graphs to display pattern data and explore a variety of ways to write rules that describe patterns and relationships; and
- develop and test generalizations based on observations of patterns and relationships.

Educational experiences in Grades 5-8 will assure that students:

- describe, analyze, create and extend a wide variety of patterns;
- represent and describe mathematical relationships using tables, rules, simple equations and graphs;
- use patterns and relationships to identify the  $n$ th term in a sequence;
- construct and analyze tables and graphs to identify patterns and relationships; and
- use patterns and relationships to represent and solve problems.

Educational experiences in Grades 9-12 will assure that students:

- identify, describe and generalize numerical and spatial patterns;
- identify, describe and generalize patterns from data and identify and analyze patterns of change; and
- predict and describe patterns produced by iterations, approximations, limits and fractals.

## CONTENT STANDARD 9: Algebra and Functions

*Students will use algebraic skills and concepts, including functions, to describe real-world phenomena symbolically and graphically, and to model quantitative change.*

### K-12 PERFORMANCE STANDARDS

Educational experiences in Grades K-4 will assure that students:

- represent numerical situations using variables, expressions, equations and inequalities; and
- write and solve number sentences that describe real-life situations.

Educational experiences in Grades 5-8 will assure that students:

- use variables, expressions, equations and inequalities to describe and represent numerical situations;
- use concrete materials, tables, graphs, verbal rules and symbolic expressions to represent situations and patterns;
- analyze functional relationships to explain how a change in one quantity is associated with a change in another;
- construct and interpret data points on number lines and the coordinate plane; and
- solve simple linear equations using concrete, informal, graphical, tabular and formal methods.

Educational experiences in Grades 9-12 will assure that students:

- model and solve problems that involve varying quantities with variables, expressions, equations, inequalities, absolute values, vectors and matrices;
- model real-world phenomena using polynomial, rational, trigonometric, logarithmic and exponential functions, noting restricted domains;
- analyze the effect of parametric changes on the graphs of functions;
- translate among and use tabular, symbolic and graphical representations of equations, inequalities and functions;
- develop, explain, use and analyze procedures for operating on algebraic expressions and matrices; and
- solve equations and inequalities using graphing calculators and computers as well as appropriate paper-and-pencil techniques.

**CONTENT STANDARD 10: Discrete Mathematics**

*Students will use the concepts and processes of discrete mathematics to analyze and model a variety of real-world situations that involve recurring relationships, sequences, networks, combinations and permutations.*

**K-12 PERFORMANCE STANDARDS**

Educational experiences in **Grades K-4** will assure that students:

- classify data according to attributes;
- solve simple counting problems;
- use diagrams and models of simple networks that represent everyday situations;
- identify and investigate sequences; and
- follow, devise and describe practical algorithmic procedures.

Educational experiences in **Grades 5-8** will assure that students:

- use systemic listing and counting strategies, including simple combinations and permutations;
- use recursive processes, including iteration, to explore and solve problems; and
- devise, describe and test algorithms for solving optimization problems.

Educational experiences in **Grades 9-12** will assure that students:

- represent problem situations using finite graphs, matrices, sequences and recurrence relations;
- develop, analyze, describe, invent and test algorithms;
- define and use permutations, combinations, mathematical induction and recursion to solve combinatorial and algorithmic problems; and
- understand and use appropriate strategies to solve optimization problems.



Curriculum  
Instructional Materials  
Instructional Technology  
Instructional Time  
Instructional Connections  
Assessment Of Students  
Professional Interaction  
Professional Development  
Professional Supervision And Evaluation  
Monitoring Programs  
Tracking And Leveling  
Remedial Assistance And Student Support  
Articulation And Alignment  
Resource Personnel And Leadership  
Administrative Understanding And Support

No matter how powerful, well designed, closely aligned with national standards, or how well written a mathematics curriculum may be, a curriculum guide is but one component of a mathematics program. In fact, to implement effective, high-performance mathematics programs, the curriculum must be embedded within a set of programmatic components that offer a coordinated design for fostering student achievement. This chapter addresses these critical components – the “program delivery standards” – that support the implementation of a curriculum that reflects what has been outlined in Chapter 2.

The following 15 distinct, yet obviously inter-related, program components that constitute a high-performance K-12 mathematics program have been identified:

- curriculum
- instructional materials
- instructional technology
- instructional time
- instructional connections
- assessment of students
- professional interaction
- professional development
- professional supervision and evaluation
- monitoring programs
- tracking and leveling
- remedial assistance and student support
- articulation and alignment
- resource personnel and leadership
- administrative understanding and support

Each of these components is presented with a statement defining the standard and a brief elaboration that describes the key elements of a program that meets the standard.■

## Curriculum

*A high-quality mathematics program is defined, guided and supported by a comprehensive, developmentally appropriate, well-written curriculum that is consistent with the vision of the National Council of Teachers of Mathematics (NCTM) standards and incorporates the Connecticut Mastery Test (CMT) mathematics objectives and the Connecticut Academic Performance Test (CAPT) mathematics framework.*

Too often, a district’s curriculum guide gathers dust in a bottom drawer. Too often, summer curriculum committee work merely codifies the table of contents of the adopted textbook. And too often, teachers are given a series of conflicting messages to “follow the text,” “follow the curriculum guide,” and “get students

ready for the test.” These problems can be overcome when a mathematics program is driven by a comprehensive, teacher-friendly curriculum guide that provides clear direction, articulation between grades and courses, and coherence among the program’s components.

A school or district’s mathematics curriculum should delineate the overarching philosophy and goals of the program, present the key objectives or outcomes for each grade level or course, provide illustrative examples, tasks and/or activities, indicate how the outcomes or objectives will be assessed, and list available resources for implementing the curriculum. In addition, the K-12 curriculum should assure smooth transition from grade to grade and course to course. In short, written curriculum guides should provide teachers (and others) with clear answers to the questions: What exactly are my students expected to learn this year? and, What skills, concepts and understandings should I be held accountable for teaching this year?

Suggestions for creating such a curriculum guide are presented in Chapter 4.■

## Instructional Materials

*A high-quality mathematics program provides each student with, and makes effective use of, appropriate instructional print materials, as well as an adequate supply of nonprint materials to accomplish the goals of the curriculum.*

Mathematics used to be the easiest of all major disciplines to support financially. Purchase one textbook for each student and an accompanying teacher’s guide for each teacher, place some blank paper, lined paper and graph paper in each classroom, ensure that each teacher has a chalkboard and an ample supply of chalk, and maybe provide an overhead projector. While these limited materials may have worked well to deliver a program limited to the mastery of arithmetic, they are insufficient to support a high-quality mathematics curriculum.

In addition to a core text or, increasingly, a set of core modules, teachers need ready access to alternative textbooks, supplemental print materials, measuring devices, geometric models, and a host of grade-level or course-appropriate manipulative materials (see Appendix E). In addition, teachers need access to an overhead projector to help present materials visually and copying machines to quickly and efficiently reproduce diverse instructional materials and assessments that support the instructional program.

Without access to such an array of nontext materials, a teacher’s ability to deliver the kind of active instruction envisioned in this guide is severely compromised.■

## Instructional Technology

*A high-quality mathematics program ensures that each student has access to necessary technological tools and makes full use of calculators and computer software to implement the goals of the curriculum.*

In every sector of our society, technology has changed how we do business, how we manufacture things, how we stay informed, and how we live our lives. Sadly, most schools lag far behind most other institutions when it comes to making full use of technology. The time has come for schools to enter the technological era and shift the fundamental delivery system of education in ways not seen since the invention of books and movable type in the 15th century.

Technology, particularly calculators that now are as commonplace as pencils, interactive probes that make algebra come alive, spreadsheets and graphing utilities that permeate the business world, and word processing capability, must be available and incorporated into the K-12 mathematics program. Just as it is inconceivable to run a bank with the ledger cards of old, or a supermarket with the manual cash registers of yesterday, it is impossible to build a high-quality mathematics program with use of just paper and pencil.

Every teacher of mathematics must recognize that technology has made some mathematics entirely obsolete (e.g., calculating cube roots), some mathematics newly accessible to students (e.g., exponential functions), and some mathematics possible for the first time (e.g., fractals). For this reason, an effective K-12 mathematics program is built on the assumptions that:

- all students have access to appropriate calculators beginning no later than Grade 4;
- all teachers have overhead projector-compatible calculators and LCD panels connected to a classroom demonstration computer; and
- all schools have site-licensed tool and utility software readily available for student and teacher use to enhance student learning of mathematics.■

## Instructional Time

*A high-quality mathematics program provides time allocations of at least 45 minutes per day at the primary level, at least 60 minutes per day at the intermediate level, and the equivalent of at least one 45-minute period per day at the middle, junior high and senior high school levels for formal mathematics instruction, supplemented less formally through interdisciplinary activities and homework.*

Time allocations – formal allocations of minutes per day, informal and interdisciplinary minutes per week, and the overall quality of how time is used – are critical variables in assuring a high-quality program. Students whose formal mathematics period is 60 minutes per day receive nearly 180 hours of instruction a year, fully 50 percent more time than students in 40-minute periods. Moreover, students who complete 20 minutes of meaningful homework four nights per week spend over 40 additional hours per school year engaged in mathematical tasks. Similarly, students in classes where the daily routine involves significant class time going over homework and starting on new homework receive far less productive instructional time than those in classes where students are actively involved exploring, investigating and solving problems.

The 45 and 60 minute-per-day recommendations represent **minimum** guidelines and are intended to ensure that schools schedule enough time for teachers and students to reasonably be able to meet the goals of the curriculum. It should be understood that the mathematics program needs to be richly supported by mathematics that is embedded in the science, social studies and vocational education curriculums. It is also expected that these allocations are supplemented through interdisciplinary and integrated tasks and units and by the ongoing, everyday use of mathematics to solve school and classroom problems. In addition, nothing in this standard should preclude the scheduling of longer blocks of time to allow students to become more deeply involved in activities and projects.

At the high school level, consideration may be given to alternative scheduling that provides for longer class periods, fewer classes per day for both students and teachers, and fewer students per teacher per semester.■

## Instructional Connections

*A high-quality mathematics program regularly makes connections both within mathematics and between mathematics and other subject areas so that students make and see the connections among the major mathematics curriculum strands and between mathematics and other disciplines.*

For too long and for too many students, learning mathematics has meant moving from topic to topic and from chapter to chapter, with little regard to the connections between and among these topics or chapters, and even less attention to the connections between the topics and their applications in the world and in other disciplines. This lack of connection results in students being forced to learn and memorize far too many bits of information without the benefit of generalizing principles or

real-world contexts that make learning easier, more enjoyable and more significant. A more inquiry-based approach tends to naturally support these instructional connections, and a more interdisciplinary approach tends to support enhanced exploration and inquiry.

One of the hallmarks of the NCTM standards is the emphasis on mathematical connections as one of the foci of all mathematics instruction. Rather than continue to view the mathematics curriculum as being composed of several discrete strands – computation, measurement, geometry, algebra – that often are taught in isolation, a high-quality program regularly fosters connections between and among these strands as well as links mathematics to everyday experiences and other disciplines.■

### Assessment Of Students

*A high-quality mathematics program has a coherent system of assessment that is closely aligned with the curricular and instructional goals of the program, and that promotes the ongoing improvement of instruction.*

In corporate America it is said that “what is inspected is respected.” Similarly, in schools, what is assessed and how it is assessed communicate most clearly what is valued. If the vision of a curriculum oriented to thinking, reasoning and problem solving is to become a reality, our system of assessment and how we hold students accountable must shift. The traditional right/wrong forms of assessment may have been appropriate for assessing the success of a rule-driven, fact-oriented curriculum, but it is increasingly clear that such forms of testing are insufficient to support the curricular and instructional changes that are needed.

Accordingly, a high-quality mathematics program must incorporate more powerful assessments of demonstrated accomplishment, using observations, performances, projects and/or portfolios. In fact, our entire system of accountability must shift from the relative standards of national percentiles and designations of percentages above and below remedial standards to more holistic judgments of student work, based on clear criteria for expected performance that hold students accountable for meeting high standards. The need to make these newer and more sophisticated judgments is one of the reasons for significant increases in various types of performance assessment like those found throughout Chapter 2 of this guide and on the mathematics subtest of the Grade 10 CAPT.

When one defines *assessment* as a process of gathering evidence about a student’s knowledge of, ability to use and disposition toward mathematics, and of making inferences from that evidence for a variety of purposes, one begins to see assessment as the third interdependent

component of a triangle: curriculum (what to teach), instruction (how to teach) and assessment (how well has learning occurred).■

### Professional Interaction

*A high-quality mathematics program ensures that teachers have ample time and diverse mechanisms to interact professionally on substantive matters of curriculum, pedagogy and assessment.*

The professional isolation of teachers is frequently cited as the most serious impediment to improved curriculum, instruction and assessment. Most teachers practice their craft behind closed doors, minimally aware of what their colleagues are doing, usually unobserved and undersupported. Far too often, teachers’ frames of reference are how they were taught, not how their colleagues are teaching. Common problems too often are solved individually rather than by seeking cooperative and collaborative solutions to shared concerns.

The magnitude of the changes that teachers of mathematics are being asked to make requires far greater opportunities for substantive professional interaction. This interaction can and must take many forms. For example, peer observations, team teaching, formal and informal opportunities for sharing, videotaping of instruction, issue-focused faculty discussions, cross-district grade-level meetings, action research teams, course committees and common planning time are all powerful vehicles for reducing professional isolation and enhancing professional interaction. Each of these strategies has a role so that school faculties and mathematics departments can become dynamic communities of learners rather than just assortments of teachers and students working in the same building or district. Each of these strategies promotes the sharing of ideas, experiences and knowledge that is the hallmark of professional collaboration.

The professional interaction described in this standard envisions teachers as collaborative leaders, not as passive followers. It envisions teachers forming study groups, conducting informal research in their classes, and engaging in the inquiry that embodies lifelong learning.■

### Professional Development

*A high-quality mathematics program is supported by a comprehensive program of professional development that focuses on issues of curriculum, pedagogy and assessment; recognizes the importance of ongoing professional growth; and provides opportunities to participate in conferences, seminars and institutes.*

In light of the magnitude of the change that teachers of mathematics are being asked to make, it must be broadly and clearly understood that it is unreasonable to expect teachers to implement these changes in curriculum, instruction and assessment when they are unaware of available programs, resources and materials, and when there is so little time for reflection on these changes. Without planned, sustained and ongoing professional development, including opportunities to attend conferences and seminars, access to professional journals, encouragement to visit with colleagues and time for reflection, teachers too often remain unaware of the paths to change and improvement. In fact, many observers of Japanese schools cite the time allocated to these forms of professional development and professional interaction as one of the starkest differences between the two systems.

For these reasons, an ongoing system of professional development for teachers of mathematics that is responsive to identified instructional needs, adequately funded and supported with sufficient time should be in place within every school and district. In addition to these professional interaction strategies, professional development opportunities scheduled for after school, weekends and summer must be supplemented with school-day released time opportunities.■

### Professional Supervision And Evaluation

*A high-quality mathematics program includes a system of professional supervision and evaluation that sets high standards of professional performance and is supported by programs and policies to assure that these standards are met.*

Just as the methods and techniques of student assessment must change to account for measuring newer and broader outcomes, so too must methods and techniques of teacher evaluation change to account for different definitions of productivity and effectiveness.

The common practice of casual and infrequent classroom observation, often carried out in a perfunctory manner, is insufficient to promote improvement in practice and, therefore, student achievement. Professional evaluation that supports a high-quality mathematics program needs to be broadened in scope and deepened in rigor. Administrators and supervisors must be better trained and more knowledgeable about effective mathematics curriculum and instruction. In addition, professional evaluation must go beyond periodic observations and include such activities as the development and peer review of professional performance portfolios, analysis of student work and student achievement, and the uses of assessment center activities like those used by the National Board for Professional Teaching Standards (see chapter references).

It is critical that classroom teachers receive constructive support – as they are supervised **and** as they are evaluated – so that excellence in teaching becomes a realistic and attainable goal for all.■

### Monitoring Programs

*A high-quality mathematics program has a set of coordinated procedures that provide ongoing monitoring and periodic evaluation to assure that student achievement goals are being met.*

Every program in every school system should be subjected periodically to careful scrutiny. Those within the system, and increasingly those outside the system, have a right to definitive answers to questions such as:

- Is the program working for all students?
- Is the curriculum meeting the needs of students and the broader society?
- Is instruction provided in ways that maximize student achievement?
- Are students achieving in sufficient numbers and at high enough levels and, if not, why?
- Are all necessary program components in place and aligned to achieve program goals?

To ensure a high-quality mathematics program, it is necessary to conduct periodic and comprehensive reviews of the entire program to answer these and other questions, publicly report findings and implement changes on the basis of the review. One source for monitoring programs is the *Connecticut K-12 Mathematics Evaluation Guide* (Connecticut Academy for Education, 1997).■

### Tracking And Leveling

*A high-quality mathematics program minimizes the leveling, sorting and tracking of students, while fully meeting the diverse and individual needs of all students.*

No single component of the educational system more powerfully communicates the expectations – both high and low – we hold for young people than the ways in which schools sort, level and track students. A major step in moving toward the vision of “mathematics for all” is a dramatic decrease in the ability-grouping, leveling and tracking of students. This does not mean abandoning all ability groups; it does not mean the elimination of all honors courses; and it does not mean all students grouped heterogeneously all of the time. It does mean, however, a change in policy at the school and district levels regarding ability-grouping and tracking so that no student is denied access to a rich and demanding mathematics program best suited to his or her individual needs and interests. In addition, such a policy recognizes

that flexible grouping and collaborative teaching can be appropriate strategies for best meeting individual needs and helping to support inclusion practices.

Stated differently, it must be recognized that the current gap in breadth, depth and rigor between what is provided for the top 20 percent and for the bottom 20 percent must be narrowed significantly by raising expectations for the bottom 20 percent. However, it must also be recognized that it is not reasonable that expectations for the top 10 percent be identical to those for the bottom 10 percent. This critical issue of levels, tracks and inclusion policies is revisited in greater depth in Chapter 5.■

## Remedial Assistance And Student Support

*A high-quality mathematics program provides supplemental instructional services to assure that all students have an opportunity to meet the goals and expectations of the program.*

While individual and diverse backgrounds, interests, learning styles or preferences, and abilities are widely recognized, schools often overlook and even ignore these differences by keeping learning time a constant. There are one-year courses, 45-minute classes, 15-minute quizzes and two-minute fact drills that apply to all students. Common sense dictates that while some students need less time, others need more time to be able reasonably to meet the goals for any lesson, unit, year or program. One way that time becomes a variable used to better meet individual student needs is through the provision of extra, remedial or compensatory instruction for those students for whom traditional time allocations are insufficient.

In addition to time adjustments, effective remedial and supplemental programs also must address adjustments in instructional methods and formats. For example, among the alternative ways that students can be better supported are with:

- teachers who possess a wide repertoire of strategies and assorted instructional materials, including hands-on materials and high-quality software, and who supplement daily instruction for needy students;
- math centers that are well stocked with materials, supplemental resources and computers, and that are staffed with trained and knowledgeable personnel;
- after-hours programs such as after-school clinics, Saturday academies and summer enrichment programs; and

- support personnel available to work with students within classrooms rather than in pull-out situations and who work closely with regular classroom teachers.■

## Articulation And Alignment

*A high-quality mathematics program evolves coherently, grade-by-grade and course-by-course, from kindergarten to Grade 12 and displays an alignment of curriculum, instructional materials, professional development and assessment that are all implemented to attain the overarching goals of the program.*

Two often-noted characteristics of what has been referred to as the “underachieving mathematics curriculum” are the mixed and often conflicting messages given to teachers about what should be taught, how it should be taught, and how it will be assessed on the one hand, and the lack of a smooth flow of mathematical content as students progress from kindergarten to Grade 12 on the other hand.

To eliminate these unwanted features, curriculum guides like those described in Chapter 4 must replace textbooks and tests as the primary driver of the program. In addition to delineating content expectations, these curriculum guides need to provide both instructional suggestions and assessment possibilities to ensure tighter alignment. Similarly, curriculum guides must be designed to ensure a developmentally appropriate sequence of outcomes, recognizing the need for exploratory exposure to mathematical ideas, opportunities to master these ideas, and time to review and reinforce these ideas. In addition, middle and high school course offerings must likewise be an articulated sequence that maximizes student readiness for each succeeding course. A common vehicle for increasing the articulation and coordination of the curriculum is through grade-level meetings of two consecutive grades and meetings of course committees where teachers share problems and concerns and make necessary adjustments.■

## Resource Personnel And Leadership

*A high-quality mathematics program assigns the responsibility for the ongoing implementation and improvement of the program to qualified coordinators, supervisors, resource personnel and/or department chairpersons in order to provide support, coordination, supervision and leadership.*

Each of the above components make it increasingly clear that a high-quality K-12 mathematics program entails a complex web of ongoing activity, change and support. Such a program cannot be put in place and be expected to run by itself. In fact, like every other aspect of a school system, the effectiveness and vitality of a district's K-12 mathematics program depends critically on the assignment of responsibility for program oversight and coordination to one or more individuals. All too often when program leadership and responsibility for ongoing implementation and improvement are not vested in one or more individuals, the focus on program quality and improvement tends to fall through the cracks and fragment under the weight of other priorities.

Because high-quality programs require that a vision of reform be created and nurtured, that teachers be kept abreast of changes and aware of professional development opportunities, and that curricular, instructional and assessment improvement are ongoing processes, it is imperative that leadership of and responsibility for the mathematics program be assigned and maintained at all levels.■

### Administrative Understanding And Support

*A high-quality mathematics program receives strong support from principals and central office administrators to ensure that these standards are met.*

Program delivery standards do not get implemented automatically or because they are listed and described in a curriculum guide. Rather, their implementation requires the understanding and tangible support of principals, curriculum coordinators, assistant superintendents and superintendents. While implementation

of a curriculum and effective delivery of instruction occurs in thousands of Connecticut classrooms, the critical support for teachers and understanding of the program described in this chapter emanate from the beliefs and actions of hundreds of Connecticut school and district administrators.

These administrators are key for setting a tone for continuous review and improvement, for maintaining continually higher expectations, and for providing teachers with the financial, material and professional development support they need to meet the school or district's objectives. Effective administrators strengthen a mathematics program by encouraging experimentation, by facilitating the review and use of school and district assessment data, and by keeping concern for student achievement in mathematics on the front burner at all times. Effective administrators also "grant permission" for teachers to take risks and shift both curriculum and instruction. In addition, administrators play a key role in encouraging and supporting teachers who are struggling to change old habits and adopt new practices.■

### References

- Connecticut Academy for Education. *Connecticut K-12 Mathematics Evaluation Guide: An Evaluation and Implementation Tool*. Middletown, CT: Connecticut Academy for Education, 1997.
- National Board for Professional Teaching Standards (NBPTS). *Adolescence and Young Adulthood/Mathematics: Standards for National Board Certification*. Detroit, MI: NBPTS, 1994.

