A Vision Of Mathematical Power And Appreciation For All 1

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Mathematics education in the United States has been bombarded by calls for change throughout the past 40 years. It has been a sometimes turbulent, often confusing period, complete with ups and downs, false starts and nearly constant debate. Whether derived from societal forces applying pressure from the outside, professional judgments arising from within, new knowledge derived from research, or technological advances that present new opportunities, **change**, and the inevitability of continued change, make the development and implementation of an effective and responsive mathematics program challenging, imperative and exciting.■

Where We've Been

A brief overview of the past 40 years demonstrates the breadth of this change and sets a context for moving into the 21st century.

The late 1950s saw the advent of the "new math" programs. Commissions and study groups were established; new projects and programs were produced. "New math" was perceived by many as being anything innovative, experimental, or simply different from the mathematics taught before. The race was on to teach more mathematical theory at lower grade levels, and to stimulate greater understanding of mathematical concepts and structure at all grade levels. These were noble goals, but fell far short in terms of implementation due in part to a lack of teacher input, inadequate training of teachers, and a paucity of public support.

The 1960s can be viewed as the age of experimentation. With the dawning of the Space Age, textbooks began to incorporate "new math" ideas and teachers were trained in the uses of new materials. Federal agencies poured money into crash programs, and new topics and approaches were assimilated into traditional curriculums. Theoretically, a reasonable and realistic plateau should have been reached, with many children well prepared in mathematics and equipped to pursue further study of more advanced topics. However, the sixties also witnessed what many skeptics warned about from the beginning: many children did not achieve at the levels demanded by the new programs. The material was too abstract for some, parents found their children's homework incomprehensible, many teachers never really bought into the new curriculum, and the pendulum began to swing again.

The 1970s brought an era of retrenchment accompanied by occasional forays into still newer areas. Out of the sixties came demands for more attention to students who were not succeeding. We entered a period of social consciousness during which emphasis shifted to the underachiever, the slow learner as well as the exceptional child. In light of declining test scores, the cry of "back-to-basics" arose. Mathematics curriculum planners returned to their drawing boards in search, once again, of a new balance between computational skills and the rest of mathematics. The seventies also witnessed the emergence of new issues, such as the role of career education in mathematics, the metric system, computer science and the proper role of calculators.

The 1980s began with the National Council of Teachers of Mathematics' (NCTM) *An Agenda for Action* and a search for resolution of the polarized debates over new vs. old, abstract vs. concrete, formal vs. intuitive. The eighties witnessed an accelerating concern for equity, a renewed focus on problem solving and applications, on developmentally appropriate curriculum and instruction, and on the inclusion of calculators throughout the mathematics curriculum. In Connecticut the 1980s began with a 9th grade Proficiency Test of basic skills and ended with a 4th, 6th and 8th grade Mastery Test of core content.

The 1990s are best characterized as an era of implementing national standards. There is little doubt that the driving force for the curricular and instructional changes advocated during the nineties arose from NCTM's *Curriculum and Evaluation Standards* (1989) and *Professional Standards for Teaching Mathematics* (1991). These documents – representing a new level of national leadership – grew out of the scattered visions of prior decades and have provided widely accepted guideposts for reform. Critical elements, such as communication and connections, tasks and discourse, alternative forms of assessment and new forms of technology, and an overriding belief in "mathematics for all" framed discussion and action throughout the nineties. In fact, this *Guide to K-12 Program Development in Mathematics* in general, and the vision for mathematics education described in this chapter, emerge in large measure from the NCTM standards.

The changes of the past 40 years have not diminished the range of issues facing the mathematics program developer. However, by reviewing the evolution of the present-day curriculum and by anticipating the future needs of our society, a strong K-12 mathematics program that is responsive to today's issues, challenges and problems can be developed.■

The Case For Continuing To Change

The world students and teachers face today is characterized by several dominant conditions:

- the widespread impact of available technology – particularly, powerful calculators and computers – on nearly every aspect of our lives;

- the changing world of work wherein mathematical ability is increasingly the key to maintaining economic viability;
- the proliferation of data and information that citizens are increasingly required to make sense of in decision making;
- the growing body of research, particularly in the field of cognitive psychology, about how students best learn and retain knowledge; and
- student achievement data from highly publicized Connecticut Mastery Test (CMT) and Connecticut Academic Performance Test (CAPT) results, National Assessment of Educational Progress (NAEP) state-by-state comparisons, and recent international comparisons where Connecticut and the United States lag behind both our needs and our expectations.

We need to broaden our understanding that the world is different and has different needs to which we are not adequately measuring up. As was noted in the 1989 National Research Council report *Everybody Counts: A Report to the Nation on the Future of Mathematics*:

> "As technology has mathematized the workplace and as mathematics has permeated society, a complacent America has tolerated underachievement as the norm for mathematics education. We have inherited a mathematics conforming to the past, blind to the future, and bound by a tradition of minimum expectations."

Alan Schoenfeld, a mathematician and mathematics educator at the University of California at Berkeley, has pointed out that, in all too many cases, the direct result of traditional instructional practices is that students:

> "acquire the view that mathematics is a mysterious domain that is accessible to only the select few, who are granted godlike status. Teachers give you rules for solving problems and exercises, which you memorize and use. These rules don't have to make sense and they probably don't, but if you do what you are told, you'll get the right answers and then everyone's happy."

These conditions raise several key questions that face the responsible and responsive developers and implementers of mathematics programs:

- How can we ensure broad mathematical literacy for all, since today nearly every worker and citizen must possess number sense, spatial sense and data sense to function productively?
- How can we ensure that our classrooms regularly exhibit the active construction of knowledge based on realistic applications, concrete materials and pictorial models?
- How can we ensure that all teachers of mathematics are properly educated, adequately supported and involved in making decisions that affect curriculum?
- How can we ensure more and better mathematics for more students in light of the accelerating pace of social and economic change – including global competition?
- How can we ensure that all students develop a sense of the value and beauty of mathematics?
- How can we ensure that all students can make appropriate use of calculators and computers to do mathematics accurately and efficiently?

Our challenge is to craft persuasive answers to these questions as we implement changes and build mathematics programs that meet today's needs and those of the future.■

Today's Mission: Mathematical Power For All

But change how? Building and implementing a high-performance mathematics program begins with a vision of mathematical content, mathematics instruction, and of the assessment of mathematical understanding. This guide and the K-12 mathematics programs it envisions, are driven by one overarching core goal: ***By the end of Grade 12, students will apply proficiently a range of numerical, algebraic, geometric and statistical concepts and skills to formulate, analyze and solve real-world problems; to facilitate inquiry and the exploration of real-world phenomena; and to support continued development and appreciation of mathematics as a discipline.*** Stated more concisely, our fundamental mission is the development of mathematical power in all students.

By *mathematical power* we mean:

- engaging in mathematical problem solving;
- reasoning mathematically;
- connecting what is learned in mathematics with other topics in mathematics, with other disciplines and with daily life;
- communicating mathematically;
- gaining confidence in one's own mathematical ability; and

- appreciating the value and beauty of mathematics.

According to NCTM, *mathematical power* "denotes an individual's capabilities necessary to explore, conjecture and reason logically, as well as the ability to use a variety of mathematical methods effectively to solve nonroutine problems. This notion is based on the fact that mathematics is more than a collection of concepts and skills to be mastered. It includes methods of investigating and reasoning, means of communication, and notions of context. In addition, for each individual it involves the development of personal self-confidence" (NCTM, 1989).

In referring to **all** students we include specifically:

- students who have been denied access in any way to educational opportunities on the basis of ethnicity, gender, socioeconomic status, handicapping condition, etc., as well as those who have not; and
- students who have not been successful or challenged in school and in mathematics, as well as those who have been successful and challenged (NCTM, 1991).

In its most simplified form, the mathematics reform being advocated here and in the national standards stems from the clarion call for a "shift in emphasis from a curriculum dominated by an emphasis on memorization of isolated facts and procedures, and proficiency with paper and pencil skills, to one which emphasizes conceptual understandings, multiple representations and connections, mathematical modeling and mathematical problem solving" (NCTM, 1989, p. 125).

Implementing this vision at the K-8 level requires that the curriculum emphasize the applications of adding, subtracting, multiplying and dividing whole numbers, decimals and fractions in the context of buying and selling, comparing, measuring, predicting and interpreting. Similarly, the curriculum at the 9-12 level must reduce emphasis on rules and procedures for manipulating symbols and increase emphasis on using and applying mathematical concepts to formulate and solve a broad range of problems that arise in diverse situations.

The vision of curriculum presented in this guide requires changes in instructional approaches as well. *Everybody Counts* says it most compellingly:

> "Evidence from many sources shows that the least effective mode for mathematics learning is the one that prevails in most of America's classrooms: lecturing and listening. Despite daily homework, for most students and most teachers mathematics continues to be primarily a passive activity: teachers prescribe, students transcribe. Students simply do not retain for long what they learn by imitation from lectures, worksheets or routine homework. Presentation and repetition help students do well on standardized tests and lower-order skills, but they are generally ineffective as teaching strategies for long-term learning, for higher-order thinking, and for versatile problem solving" (NRC, 1989).

Our vision for mathematics instruction involves classrooms where students are regularly engaged in sustained work, inquiring about and working to make sense of mathematical ideas, and constructing personal meaning of these ideas. Instead of learning solely by remembering, students are expected to "use what they are taught to modify their prior beliefs and behavior, not simply to record and store what they are told. It is students' acts of construction and invention that build their mathematical power and enable them to solve problems they have never seen before" (NRC, 1989).

Accordingly, the NCTM standards and this guide encourage four critical shifts:

- In curriculum, a shift toward a deeper study of mathematical ideas and concepts and their uses in today's world;
- In learning, a shift toward more active student involvement with mathematics, including mathematical problems that relate to their world and the use of a variety of mathematical tools for solving these problems;
- In teaching, a shift toward creating classrooms that offer stimulating learning environments in which all students have an opportunity to reach their full mathematical potential; and
- In assessment practices, a shift toward student evaluations that are continuous and based on many sources of evidence (NCTM, 1994).

When these shifts have taken place, it will be easy to see the difference in the following areas:

Classrooms no longer are passive environments with the teacher at the chalkboard or overhead projector talking and with the students listening and doing exercises based on the lecture. Direct teaching is still used, but so are other approaches. The desks are not always facing front because students may be clustered at times in small teams to work together, as is done in business and industry, to solve problems or complete mathematical projects. There are computers around the room used by some students, and most students have calculators.

The room will be a little noisier than usual, but it is purposeful noise – the sounds of students involved in mathematical learning.

Teachers lecture at times, but no longer are they always in the front of the room. Depending on the mathematical task, teachers may walk around the room, helping teams when they have questions, working individually with students and, when necessary, clarifying ideas and concepts at the chalkboard, overhead projector or computer to small groups of students. In this role teachers are coaches, responsible for selecting and orchestrating tasks, setting high expectations, and creating a classroom environment in which high-quality mathematics learning can flourish.

Students take greater responsibility for their own learning and are challenged to meet higher expectations. In or outside classrooms, they engage in important mathematical tasks while interacting with the teacher, instructional materials and equipment, and each other. Students can be seen struggling at times, but also basking in a sense of accomplishment as they share their thinking and their work with classmates. Students begin to see mathematics as a living discipline that they can understand, rather than a collection of rules they have to memorize for a test and then forget.

Tasks in which students are engaged are based on sound and significant mathematics and encourage students "to reason about mathematical ideas, to make connections, and to formulate, grapple with and solve problems. Good tasks test skill development in the context of problem solving, are accessible to students, and promote communication about mathematics" (NCTM, 1991).

Homework assignments are more than pages of exercises where students repeat the same process over and over. Instead, homework engages students in problems that relate to their own lives, interests and environments, particularly problems that require the application of the skills and concepts that were studied in class or that involve collecting data and other information that will be used in class.

Tests and grades no longer come only at the end of a unit of learning. Depending on the results, teachers may decide to clarify certain portions of the material that were not well understood. Moreover, judgments about students' learning are not based solely on a few timed paper-and-pencil tests; neither are grades. Instead, course grades reflect performance on projects, tests, classwork and homework to furnish a more complete picture of students' understanding (adapted from *Making A Living, Making A Life*, NCTM, 1994).■

Core Beliefs And Key Questions

This vision is obviously predicated on a set of core beliefs about learning. It is important to make these beliefs explicit, as they constitute the heart of effective instruction and the overarching philosophy upon which the mathematics guide and framework are based. These core beliefs include the following:

- Learning is maximized when teachers focus on thinking, imagining, reasoning, intuiting, questioning, creating, proving and other higher-order processes.
- Learning occurs best when instruction is active and engaging, and involves thought-provoking work.
- Learning is enhanced in a dynamic and collaborative school culture that encourages reflection, analysis and risk-taking.
- Learning occurs best in school environments that recognize and value the wide variety of learning styles and the collective strengths that individual students bring to the classroom.
- Learning is maximized when content is placed in context and is connected to other content, and when students are provided with multiple opportunities to construct their understanding.
- Learning is most effective when discrete skills and subject-matter content are not ends, but means toward the solution of meaningful problems.
- Learning is enhanced when all areas of the curriculum focus on concepts and applications, and when the recall of facts, rules and procedures is not used to deny access to reasoning and problem-solving activities.

Armed with this vision and with the suggestions, illustrations and examples that make up this guide, Connecticut educators who are concerned about improving the quality of mathematics programs will better be able to respond to the following guiding questions:

- Are we teaching the mathematics that our students need in today's world and will need in the future?

- Are we teaching mathematics in a way that will develop students who possess the mathematical power that permits them to use mathematics productively?
- What, in the final analysis, does it mean to be mathematically literate in a world that relies on calculators and computers to carry out mathematical procedures and a world where mathematical knowledge is rapidly growing and being applied in so many fields?

This guide provides Connecticut's best answers to these questions.■

References

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