CONNECTICUT'S MATHEMATICS FRAMEWORK 2

Program Goals
K-12 Content Standards
Grade Cluster Performance Standards
Illustrative Tasks and Experiences
Prototype Assessments
Samples of Student Work

This chapter describes the content of a K-12 mathematics curriculum for all students. It begins with a statement of the overarching goal for school mathematics and with seven K-12 mathematics program goals that should be reflected throughout the program and that should be embodied in each of the instructional tasks and activities conducted with students. Then, 10 K-12 mathematics content standards define expectations for 10 relatively distinct domains of mathematical understanding.

Each content standard, e.g., number sense or measurement, is then elaborated upon with the following three components:

- **grade cluster performance standards** for Grades K-4, 5-8 and 9-12 that answer the question, "What mathematics should all students know and be able to do?" These standards are adaptations of the National Council of Teachers of Mathematics (NCTM) curriculum standards and specify the mathematical skills, concepts and understandings that all students should be able to demonstrate by the end of Grades 4, 8 and 12 respectively.
- **illustrative tasks and experiences** that answer the question, "What should this mathematics look like in the classroom?" These illustrative tasks represent a broad cross section of the kinds of tasks and activities in which students should regularly be engaged. The tasks vary in difficulty, breadth and richness. However, all are designed to show how instruction can embed the core program goals into day-to-day teaching of mathematical content. Moreover, while the tasks have been designated as K-4, 5-8 or 9-12, it will be obvious that some of the K-4 tasks, for example, could be easily adapted for use in Grades 5-8 and vice versa. For this reason, the user of this guide is encouraged to review the entire set of illustrative tasks and experiences for each content standard.
- **prototype assessments and samples of student work** that answer the question, "How do students demonstrate their understanding?" These prototype assessments are designed to provide a tangible sense of how best to assess student mastery of the curriculum. These diverse assessment tasks reflect a blurring of the line between instruction and assessment and reflect the belief that students should be learning while they are assessed, as well as assessed while they are learning. The student work for the prototype assessments provides concrete examples of what should be expected from students. These exemplars represent full and complete work and serve to remind us that what our students actually do is far more important than the goals and standards we simply put on paper.

In using the mathematics framework outlined in this chapter to design a K-12 mathematics curriculum, it is important to remember that the content described is recommended for **all** students. That is, this is a core curriculum that we believe all students can attain – some with more time, others with alternative approaches, and some who will far exceed what is outlined in this chapter.

It is critical that users of this chapter remember that the seven program goals, including communicating, reasoning, and selecting and using appropriate tools and approaches, are expected to be embedded in **all** instructional activities. There is no problem-solving content standard because problem solving is expected to be integral to all instruction. Similarly, there is no content standard on applications or on decision making because it is expected that these and other broader goals are embedded throughout the curriculum.

It is also important to understand that, although the curriculum has been subdivided into 10 separate standards or domains, nearly every illustrative task and prototype assessment crosses content lines and either fos-

ters understanding or assesses understanding of mathematics in more than one area. This integration, inevitable when teachers use a problem-based approach, helps to develop the critical connections between and among the 10 standards.

Finally, it should be noted that, while the entire curriculum framework is designed to answer the question, "What mathematics should all students know and be able to do?," embedded in this chapter is Connecticut's answer to the broader question, "Why is mathematics in the curriculum?" One needs to look no further than today's worlds of work, citizenship and of daily living to see that we are:

- bombarded with **data** that must be analyzed and used to make decisions;
- surrounded by **change** that must be put into mathematical form to be understood and from which predictions can be made;
- forced to deal with the **ambiguity** and **uncertainty** that exist in many situations; and
- confronted in countless situations with a dazzling array of **patterns** from which we try to draw conclusions.

It is these data, changes, ambiguities, uncertainties and patterns that form the language of mathematics and that explain the prominent place in the overall curriculum that mathematics has rightly attained. This curriculum guide has been designed to prepare students to face these data, changes, ambiguities, uncertainties and patterns with competence and confidence.

MATHEMATICS

By the end of Grade 12, students will apply proficiently a range of numerical, algebraic, geometric and statistical concepts and skills to formulate, analyze and solve real-world problems; to facilitate inquiry and the exploration of real-world phenomena; and to support continued development and appreciation of mathematics as a discipline.

PROGRAM GOALS

As a result of education in Grades K-12, students will:

- communicate numerical, geometric, algebraic and statistical ideas orally and in written form with models, pictures, graphs and mathematical symbols, using paper and pencil, a variety of calculator displays, spreadsheets, graphing packages, word processing and other related computer software;
- use inductive and deductive reasoning to make, defend and evaluate conjectures and arguments, to justify assertions and verify tentative conclusions, and to solve mathematical problems;
- use mathematical skills and concepts to make and justify decisions and predictions, to identify patterns and trends, to pose questions from data and situations, and to formulate and solve problems;
- identify and use connections within mathematics to identify interrelationships and equivalent representations, to construct mathematical models, and to investigate and appreciate mathematical structure;
- use mathematical skills and concepts to describe and analyze data and measurements from other disciplines;
- select and use appropriate approaches and tools for solving computational, geometric and algebraic problems, including estimation, mental computation, guess and test, paper and pencil, calculators and computers with software for tabulating, charting, graphing, drawing and transforming data and images; and
- use mathematical skills and concepts with proficiency and confidence, and appreciate the power and utility of mathematics as a discipline and as a tool for solving problems.

K-12 CONTENT STANDARDS

1.	**Number Sense**	Students will use numbers to count, measure, compare, order, scale, locate and label, and use a variety of numerical representations to present, interpret, communicate and connect various kinds of numerical information.
2.	**Operations**	Students will add, subtract, multiply and divide with whole numbers, fractions, decimals and integers, and develop strategies for selecting the appropriate computational and operational methods for solving problems.
3.	**Estimation and Approximation**	Students will make estimates and approximations, and judge the reasonableness of results.
4.	**Ratios, Proportions and Percents**	Students will use ratios, proportions and percents to represent relationships between quantities and measures and solve problems involving ratios, proportions and percents.
5.	**Measurement**	Students will make and use measurements in both customary and metric units to approximate, measure and compute length, area, volume, mass, temperature, angle and time.
6.	**Spatial Relationships and Geometry**	Students will analyze and use spatial relationships and basic concepts of geometry to construct, draw, describe and compare geometric models and their transformations, and use geometric relationships and patterns to solve problems.
7.	**Probability and Statistics**	Students will use basic concepts of probability and statistics to collect, organize, display and analyze data, simulate events and test hypotheses.
8.	**Patterns**	Students will discover, analyze, describe, extend and create patterns, and use patterns to describe mathematical and other real-world phenomena.
9.	**Algebra and Functions**	Students will use algebraic skills and concepts, including functions, to describe real-world phenomena symbolically and graphically, and to model quantitative change.
10.	**Discrete Mathematics**	Students will use the concepts and processes of discrete mathematics to analyze and model a variety of real-world situations that involve recurring relationships, sequences, networks, combinations and permutations.

CONTENT STANDARD 1: Number Sense

Students will use numbers to count, measure, compare, order, scale, locate and label, and use a variety of numerical representations to present, interpret, communicate and connect various kinds of numerical information.

K-12 PERFORMANCE STANDARDS

Educational experiences in **Grades K-4** will assure that students:

- use real-life experiences, physical materials and technology to construct meanings for whole numbers, commonly used fractions and decimals;
- understand our numeration system by modeling, counting, grouping and using place-value concepts;
- use numbers to count, as measures, labels and as indicators of location;
- use models and pictures to demonstrate understanding of equivalent forms of numbers;
- understand and use properties of numbers, including odd, even, ordinal and cardinal; and
- develop a sense of magnitude of numbers by ordering and comparing whole numbers, commonly used fractions, decimals and money amounts.

Educational experiences in **Grades 5-8** will assure that students:

- use real-life experiences, physical materials and technology to construct meanings for whole numbers, commonly used fractions, decimals and money amounts, and extend these understandings to construct meanings for integers, rational numbers, percents, exponents, roots, absolute value and scientific notation;
- model, represent and use numbers in a variety of equivalent forms (integer, fraction, decimal, percent, exponential and scientific notation) as they arise from real-world situations;
- use the equivalence of fractions, decimals and percents to select appropriate and efficient ways to write, order, compare, estimate and compute;

(continued)

Educational experiences in **Grades 9-12** will assure that students:

- use real-life experiences, physical materials and technology to construct meanings for rational and irrational numbers, including integers, percents and roots;
- use number sense and the properties of various subsets of real numbers to solve real-world problems;
- develop and use an intuitive sense of the magnitude of numbers (including very large and very small numbers) and relate them to place value and exponential forms; and
- select an appropriate form to represent and use numerical data (integer, fraction, decimal, ratio, percent, exponential, scientific notation, irrational, complex) as they arise from real-world situations involving magnitude, order, measures, labels, locations and scales.

K-12 PERFORMANCE STANDARDS, continued

Educational experiences in **Grades 5-8** will assure that students:

- develop and use a sense of order and magnitude of fractions, decimals, integers, powers and roots; and
- develop and apply number theory concepts (primes, factors, multiples and divisibility rules), as appropriate, in various real-world problem situations.

ILLUSTRATIVE TASKS AND EXPERIENCES

As part of ongoing mathematics instruction in Grades K-4, students should have instructional experiences like the following:

1. Fast Food And Different Meals

Provide each student with a real fast-food establishment menu, including items and prices. Ask students to determine whether or not each of the students in your class can order a **different** meal for lunch according to the following rules:

- Each student has only $4 to spend.
- Each student must order at least two items.
- One choice must be a main entree.
- Any money that is not spent goes back to the teacher.

Ask students to select and compute the price of their meal. Record each meal, assuring no duplication of orders, and determine whether there is enough variety for each student to order a different meal.

Extensions: Construct a graph showing the costs of the different meals. Use organized lists to determine exactly how many unique meals can be created using different rules.

2. Bear's Breakfast

Explain to students that bears usually live in the wilderness, including some of the national parks of North America. Except during their long winter hibernation, bears eat a lot. In fact, they eat practically all the time!

Tell students that the typical North American brown bear eats about 9,000 calories for breakfast. Use the following menu to make a plan for the bear's "breakfasts" for a week. Include at least three different kinds of food for each meal. Remember, bears like variety! Explain why your plan is reasonable.

THE BEAR'S BREAKFAST MENU

Food	Calories
Beetles (1 pound)	872
Bullfrog (1 pound)	372
Fish (whole, about 3 pounds)	1920
Honey (1 pound)	1379
Snake (1 pound)	427
Termites (1 pound)	1616
Rabbit (2 pounds)	1470
Raspberries (10)	20
Red ants (1 pound)	59
Strawberries (10)	60
Turtle eggs (about 15 eggs)	673
Worms (1 pound)	1000

3. "Smart"

Distribute copies of Shel Silverstein's poem "Smart" to students.

SMART

My dad gave me one dollar bill
'Cause I'm his smartest son,
And I swapped it for two shiny quarters
'Cause two is more than one!

And then I took the quarters
And traded them to Lou
For three dimes - I guess he don't know
That three is more than two!

Just then, along came old blind Bates
And just 'cause he can't see
He gave me four nickels for my three dimes,
And four is more than three!

And I took the nickels to Hiram Coombs
Down at the seed-feed store,
And the fool gave me five pennies for them,
And five is more than four!

And then I went and showed my dad,
And he got red in the cheeks
And closed his eyes and shook his head-
Too proud of me to speak!

(From: Shel Silverstein's, *Where the Sidewalk Ends.*
Copyright 1974 by Evil Eye Music, Inc.
Used by permission of Harper Collins Publishers.)

Describe what happens with the money after each trade.
Was the father proud of his child? Why, or why not?

4. Place The Digits Game

Explain the rules for the "Closest to 100 Game" to students and model a game with the class.

"Closest to 100"

Object: To come as close to 100 as possible, without going over.

Gameboard: A large T-frame for each player, labeled "tens" on the left and "ones" on the right.

Each game consists of seven turns for each player, where each turn consists of one roll of one 1 to 6 number cube.

Rules: Take turns rolling the number cube and recording your number under the tens or the ones column. For example, if you roll a 5, it can be recorded as 5 in the ones column or 5 in the tens column (to represent 50). Keep a running total and make decisions about placing each digit based on your current total and the chance of different numbers coming up. The game ends when each player has had seven turns. The winner is the player whose total is the closest to 100 without going over.

Extensions: After playing the game several times, devise a written strategy that you might use to win the game. Increase the size of the target number to incorporate three or four places. Use subtraction from a given number and let the winner be the player closest to zero.

As part of ongoing mathematics instruction in Grades 5-8, students should have instructional experiences like the following:

1. One Million

Use the book *How Much is a Million?* by David M. Schwartz and Steven Kellogg (William Morrow & Co., Inc., 1993), and ask students to solve a range of problems involving 1,000,000. For example:

- How old is a person 1,000,000 seconds old? 1,000,000 minutes old? 1,000,000 hours old? 1,000,000 days old?
- How high would a stack of one million $1 bills be? How big a box would be needed to store one million $1 bills?
- How much space would be needed if 1,000,000 people attended an outdoor concert?
- After completing these activities, explain how your concept of 1,000,000 has changed?

2. A Timely Power Failure

Set the problem context for students by explaining that a woman returned home from work one day last week to find that the power had been off sometime during the day. She has two clocks in the bedroom – a digital clock which resets to 12:00 when the power comes back on and a standard electric alarm clock with hands that stop moving when the power goes off and start moving again when the power returns. The digital clock showed 8:21. The "hands" clock showed 4:58. Her wrist watch showed 5:56. Use the information presented to determine and explain:

- What time did the power go off?
- What time did the power go back on?
- How long was the power off?
- Why might this information be important or helpful?

3. Hot Dog Sales

Ask students to imagine that each is in charge of the hot dog stand at a high school football game. Tell students that:

- the stands can seat 1,680 people and the game is a sellout;
- about one of every three people buys a hot dog; and
- you can charge 95 cents for each hot dog you sell.

In addition, tell students that the grocery store sells:

- hot dogs in packages of 10 for $3.29;
- hot dog buns in packages of eight for 88 cents;
- jars of mustard for $1.12 each (one jar needed per 40 hot dogs sold); and
- jars of relish for $1.75 each (one jar needed per 60 hot dogs sold).

Ask students to write a report to the Booster Club detailing how much profit they can expect to make at each sold-out game from the hot dog stand.

4. Paper Problem

Provide students with the following two pieces of information:

- A recycling guide states that newspapers weigh about 58 pounds per cubic foot.
- An Ad Council ad for Project Earth states that every ton of paper not sent to the landfill saves about three cubic yards of waste.

Ask students to explain whether or not both of these statements can be true and what assumptions they would make to help them decide. If you find out that the average daily newspaper weights about one-half a pound and that the recycling center sells recycled newspapers for 2 cents per pound, ask students to calculate how much money their local recycling center can expect to make from the sale of recycled newspapers and explain how they arrived at their answer. Extend this to other local recycling issues in your community and the costs and savings involved.

As part of ongoing mathematics instruction in Grades 9-12, students should have instructional experiences like the following:

1. Counting To One Million

Ask students to estimate how long they think it would take to count aloud to one million. Record the estimates and begin a discussion of how the time required for such a feat can be accurately estimated. The discussion might include the fact that some numbers require more time to say than others. Ask students to collect data by recording the time required to say five consecutive numbers of different lengths and to prepare a table showing the number of digits and the time it takes to say five such numbers. Ask students to recalculate the time and to describe the approach they used, including any graphs, data and equations. Extend the problem by predicting the time it would take to count to one billion, one trillion and to the number of dollars in the federal debt.

2. Exponents

Ask students to use the guess and check strategy and a calculator to solve the following equation for x: $2^{15} = 10^x$. First estimate a value for x and then explain how you arrived at a solution.

3. Sturbridge

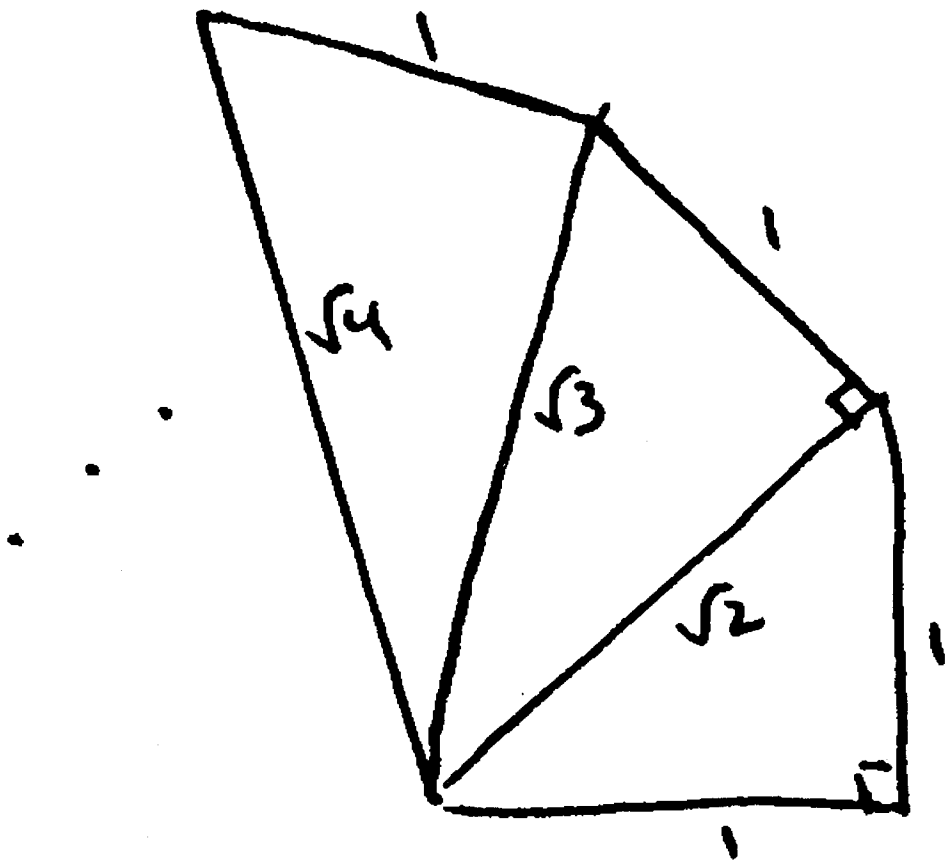
Describe the following situation for students: The landfill site and adjacent recycling center in Sturbridge, Mass. is a model for the country. Papers, tires, glass, plastics, aluminum cans and used oil all get recycled. The result is that only 40 cubic yards of compacted garbage are discarded in the landfill site each week. In 1995, the population of Sturbridge was 8,563 people. A cubic foot of compacted garbage weighs about 70 pounds. Based on this situation, ask students to decide whether Sturbridge residents discard more or less garbage than the national average of 4.1 pounds per person each day? (Note: This problem can be reworked for a community in Connecticut.)

From *Sturbridge: Woodstock. Growth in the News.*
Adapted and used with permission from COMAP.

4. Constructing Irrational Numbers

Use a ruler and protractor to construct a "square root spiral" beginning with a 1-inch by 1-inch isosceles right triangle (with a hypotenuse of $\sqrt{2}$). Then construct a 1-inch-long perpendicular to the hypotenuse at one end of the hypotenuse to create a second right triangle with sides of 1, $\sqrt{2}$ and $\sqrt{3}$. Continue constructing 1-inch-long perpendiculars that become legs of each new triangle, thereby creating a spiral of hypotenuse lengths of 1, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$, etc. Continue to $\sqrt{12}$ and then use a ruler to estimate the value of each of the radicals.

Extension: If $\sqrt{2}$ is approximately 1.41 and $\sqrt{3}$ is approximately 1.73, approximate the values of $\sqrt{6}$, $\sqrt{8}$ and $\sqrt{12}$ without measuring, thus checking some of your estimates.



PROTOTYPE ASSESSMENTS AND SAMPLES OF STUDENT WORK

As a result of an instructional program in mathematics like that described in this guide, by the end of Grade 4, all students should be expected to complete work like the sample below.

HOT DOG BUNS

Your job for your class picnic is to bring the hot dog buns. Your class estimates that you will need at least 40 buns. The store sells hot dog buns in packages of 8 and in packages of 12.

HOT DOG BUNS
1 package of 8 buns - $1.00
1 package of 12 buns - $1.20

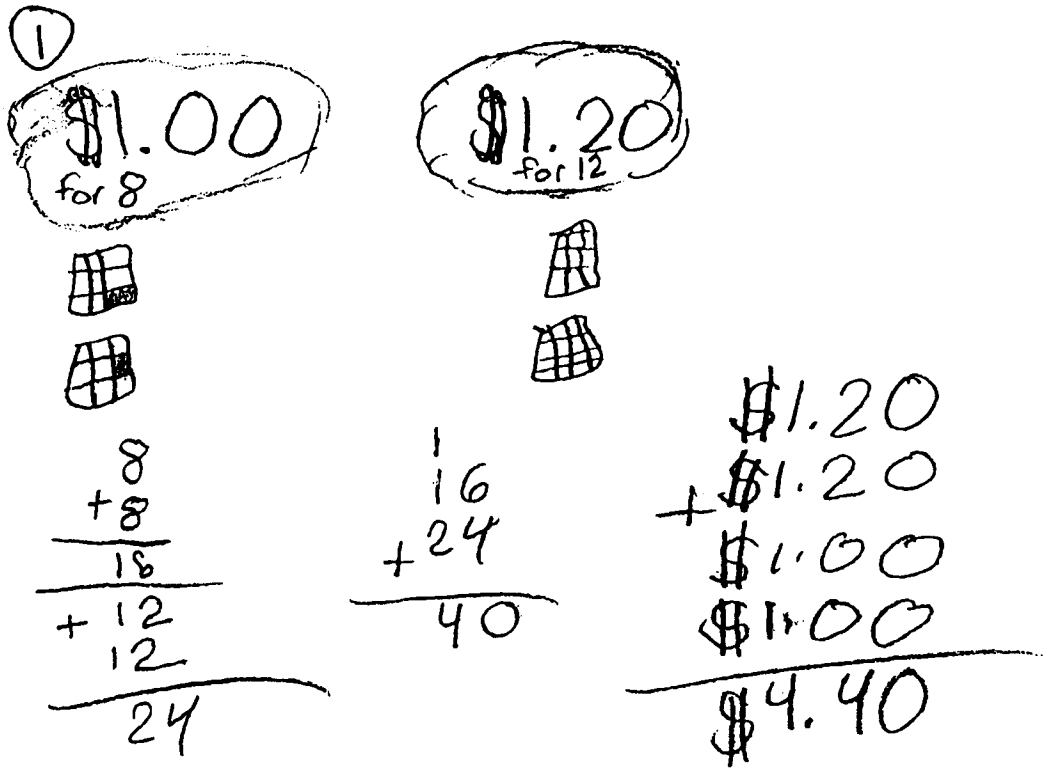
You will have to buy several packages and may end up with some extra buns.

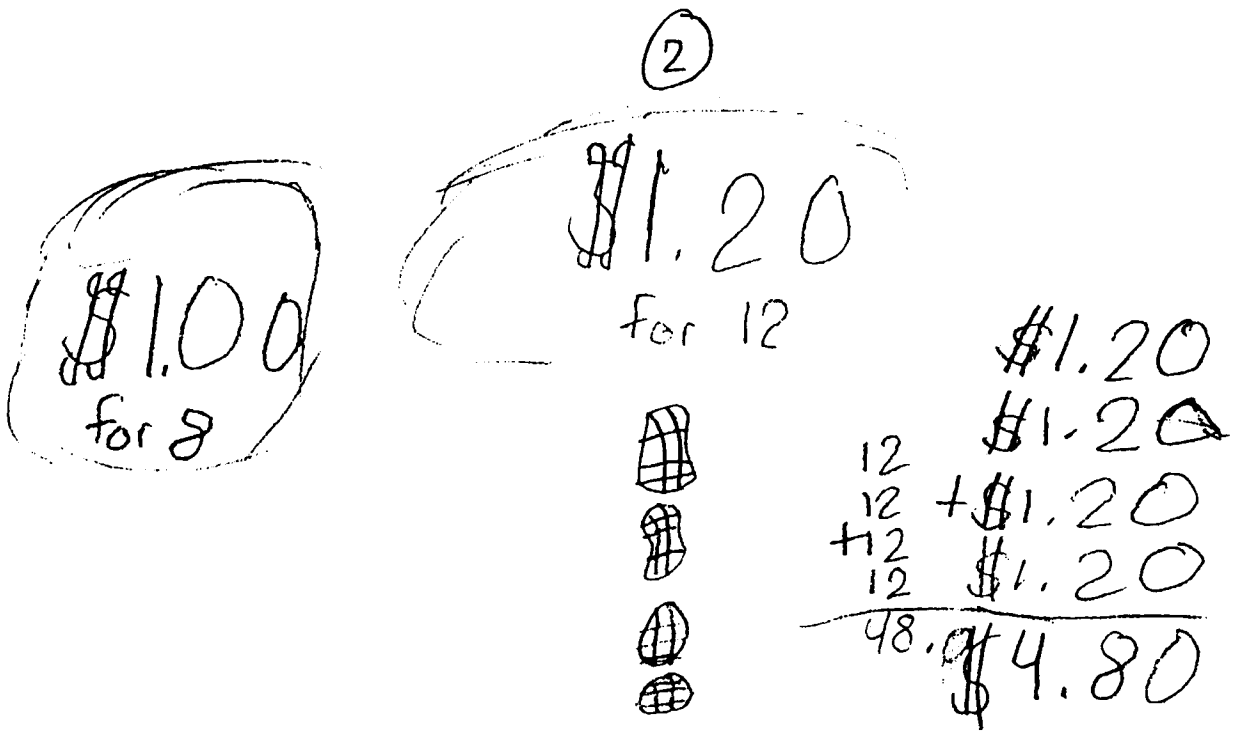
Task 1: Show three different combinations of hot dog bun packages that you could buy for the picnic. Show the cost of each combination of packages.

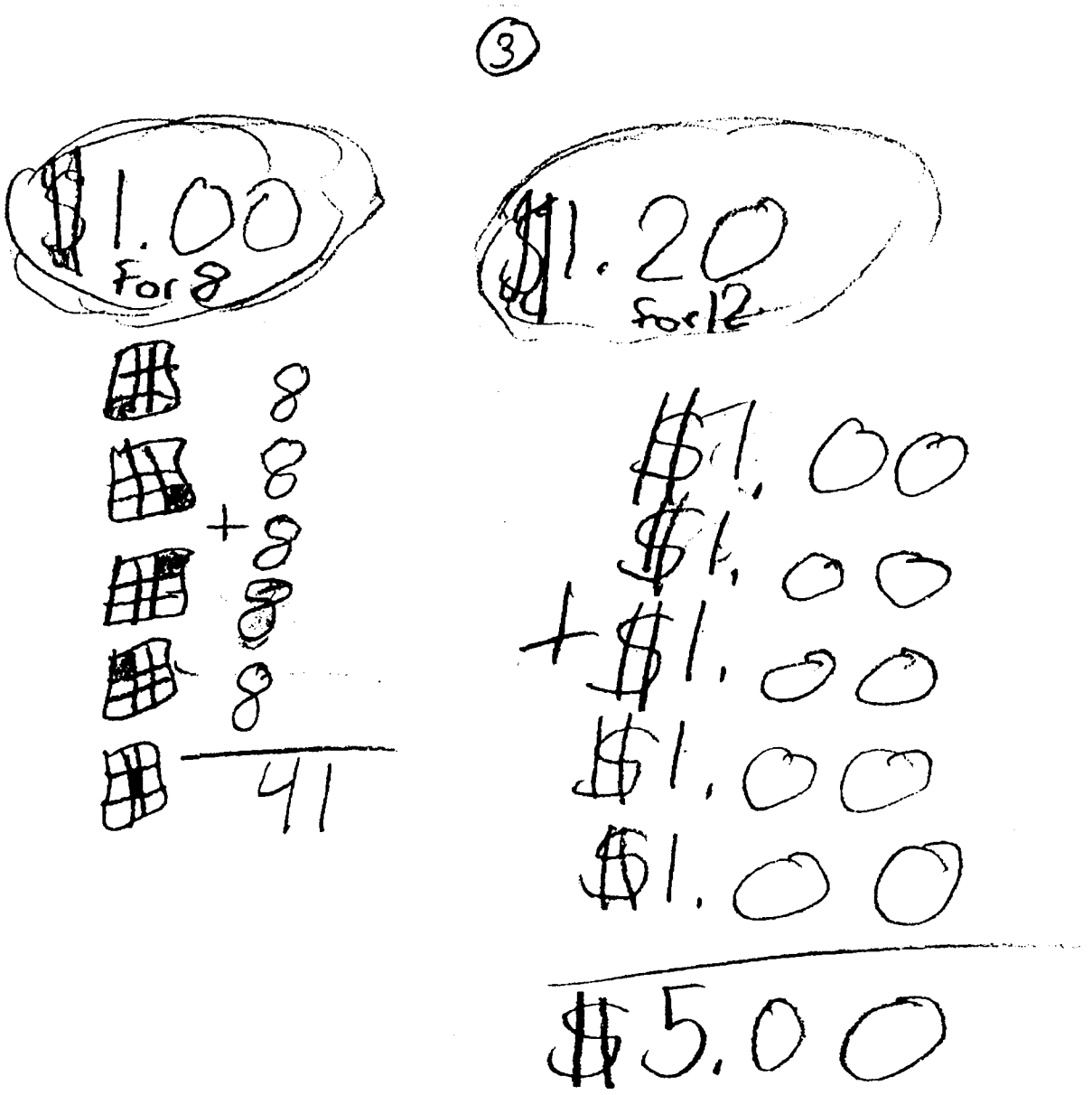
Task 2: Which combination of packages would have the fewest buns left over?

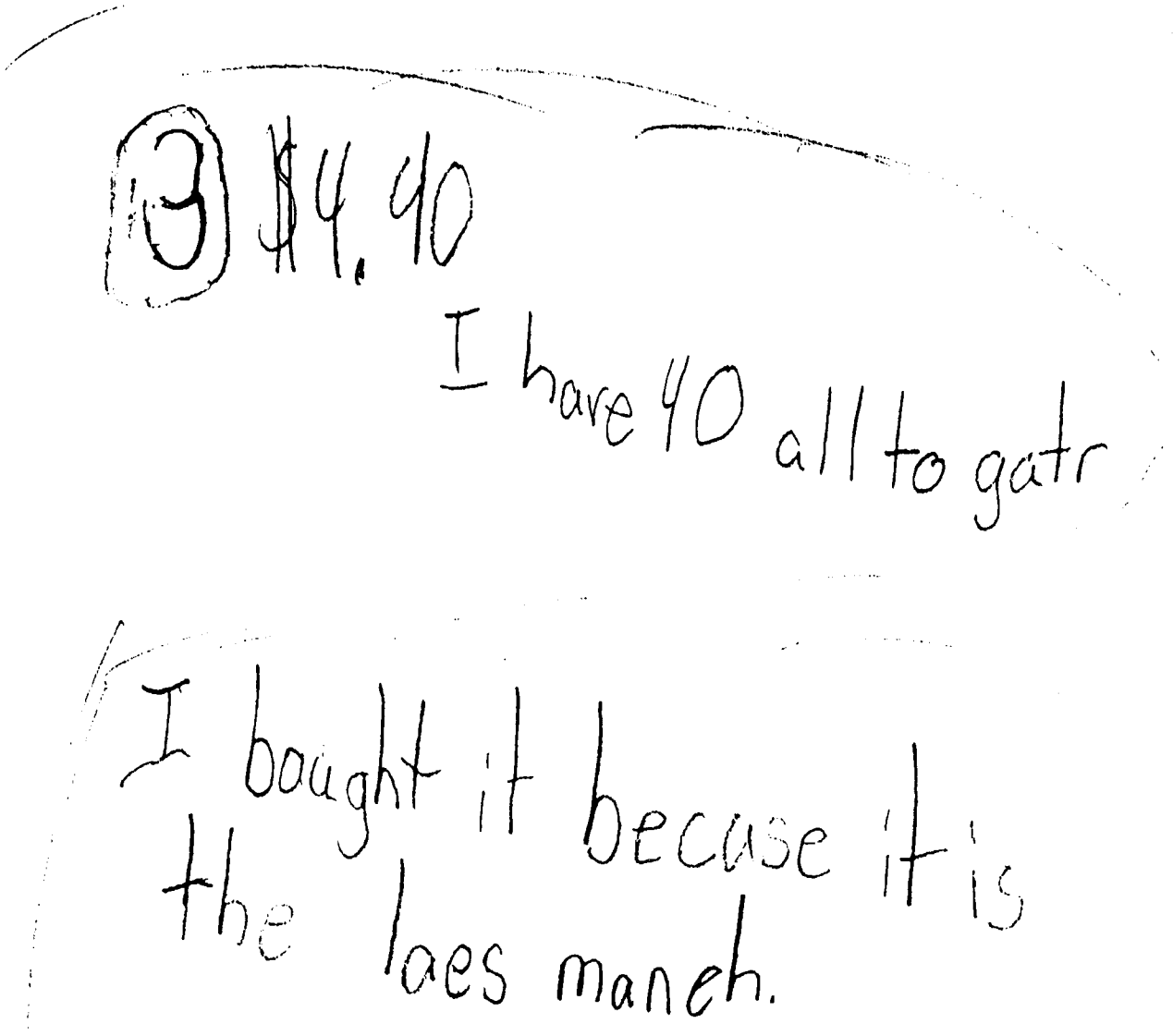
Task 3: Which combination of packages costs the least amount of money?

Task 4: Which combination of packages would you actually buy? What is your reason for this decision?


1
$1.00
for 8
$1.20
for 12
8
+8
16
+12
12
24
1
16
+24
40
$1.20
+$1.20
$1.00
$1.00
$4.40


2
$1.00
for 8
$1.20
for 12
$1.20
$1.20
12
12
+12
12
48
+$1.20
$1.20
$4.80


3
$1.00
for 8
8
8
+ 8
8
8
41
$1.20
for 12
$1.00
$1.00
+ $1.00
$1.00
$1.00
$5.00


(3) $4.40
I have 40 all to gatr
I bought it because it is
the laes maneh.

As a result of an instructional program in mathematics like that described in this guide, by the end of Grade 8, all students should be expected to complete work like the sample below.

THE DELI

The prices of some commonly purchased deli items are shown below:

Balogna	**$2.59/pound**
Salami	**$2.29/pound**
American Cheese	**$1.89/pound**
Turkey Breast	**$3.49/pound**

1. You need three-quarters of a pound of salami and a half-pound of turkey breast. You have only $6 with you. Assuming there is no tax on these items, explain how you know you do or do not have enough money for the salami and the turkey.

2. You order 1 3/4 pounds of American cheese. The clerk slices the cheese, puts it on the scale and the scale shows 1.34 pounds. Explain what you would say to the deli clerk to be sure you get the right amount of cheese.

3. Your grandmother is on a strict diet that limits her to 3 ounces of cheese per day. You know that there are 20 slices of American cheese in one pound and that there are 16 ounces in a pound. Write what you could say to your grandmother to explain to her exactly how many slices of cheese she could eat each day.

√x

(1) $2.29 \times \frac{3}{4} = \frac{2.29}{1} \times \frac{3}{4} = \frac{6.87}{4} = 1.71$

$2.29 \times 3 = 6.87$

$4 \overline{)6.87}$ → 1.71

$3.99 = \frac{3.50}{1} \times \frac{1}{2} = \frac{3.50}{2} = 1.75$

$2 \overline{)3.50}$ → 1.75

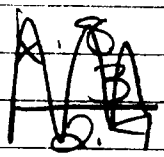
$1.75 + 1.71 = 3.46$

I would have enough money cause ~~when you add $1.71 and $1.75 you~~ get $3.46 and since I have $3.60 so I would have extra money.

(2) They ripped me off cause 3/4 as a decimal is .75, so 3/4 is 1.75 not 1.34

3. $16 \div 20 =$ $20\overline{)16.0}$ (.8) $\quad 160 \quad 0$

.8 ounces in one slice



$\times .8$
3
2.4

~~I told my grandmother to eat two and half pieces cheese cause .8 ounces in one piece of cheese~~

I told my grandmother that she can eat 3 pieces of cheese cause when you multiply .8 and 3 you get 2.4 and that is close enough to 3 ounces with out going over.
.8 means that there is .8 ounces in a piece of cheese and my grandmother is only able to eat 3 ounces so when you do 3 x .8 you get 2.4 ounces but if you do 4 x .8 you get 3.2 ounces and that is to much.

As a result of an instructional program in mathematics like that described in this guide, by the end of Grade 12, all students should be expected to complete work like the sample below.

CHEAP GAS?

On a recent trip to Europe, Khayree and Alicia observed the following gasoline prices:

Spain – 5 pesos per liter
France – 4 francs per liter
Germany – 3 Deutschmarks per liter
Denmark – 2.8 Kroner per liter
England – 0.4 English pounds per liter

Khayree said: "Boy! Gas is so much cheaper here than back home in the United States. This is really great!"

Alicia replied: "Khayree, you're nuts! It's way more expensive here than in the U.S."

Who was correct? Use the tables below to help you decide. Show all of your work. For purposes of comparison, the average price of gasoline in the U.S. is $1.30 per gallon.

CURRENCY CONVERSION TABLE

$1.00 = 5 pesos
$1.00 = 4.5 francs
$1.00 = 2 Deutschmarks
$1.00 = 2.2 Kroner
$1.00 = 0.67 English pounds

LIQUID MEASURE CONVERSION TABLE

1 quart = 0.94 liters
1 gallon = 4 quarts

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Cheap Gas?

First it is necessary to convert all the measurements to the same scale. For convenience, let us change all the gasoline prices into dollars per gallon, to compare them to the price in the United States.

Spain $\frac{5 \text{ pesos}}{\text{liter}} \times \frac{\$1}{5 \text{ pesos}} \times \frac{.94 \text{ lit.}}{1 \text{ qt}} \times \frac{4 \text{ qts.}}{1 \text{ gal}} = \$3.76/\text{gal}$

France $\frac{4 \text{ francs}}{1 \text{ lit}} \times \frac{\$1}{4.5 \text{ fr.}} \times \frac{.94 \text{ lit}}{1 \text{ qt}} \times \frac{4 \text{ qts}}{1 \text{ gal}} = \$3.34/\text{gal}$

Germany $\frac{3 \text{ Deut.}}{1 \text{ lit}} \times \frac{\$1}{2 \text{ Deut.}} \times \frac{.94 \text{ lit}}{1 \text{ qt}} \times \frac{4 \text{ qts}}{1 \text{ gal}} = \$5.64/\text{gal}$

Denmark $\frac{2.8 \text{ Kro.}}{1 \text{ lit}} \times \frac{\$1}{2.2 \text{ Kr.}} \times \frac{.94 \text{ lit.}}{1 \text{ qt}} \times \frac{4 \text{ qts}}{1 \text{ gal}} = \$4.79/\text{gal}$

England $\frac{£.4}{1 \text{ lit}} \times \frac{\$1}{£.67} \times \frac{.94 \text{ lit}}{1 \text{ qt}} \times \frac{4 \text{ qt}}{1 \text{ gal}} = \$2.24/\text{gal}$

We can see from the converted values that the price of gasoline in the United States is, indeed, cheaper than in Europe.

CONTENT STANDARD 2: Operations

Students will add, subtract, multiply and divide with whole numbers, fractions, decimals and integers and develop strategies for selecting the appropriate computational and operational methods for solving problems.

K-12 PERFORMANCE STANDARDS

Educational experiences in **Grades K-4** will assure that students:

- develop meaning for the operations by modeling, comparing and discussing a variety of problem situations;
- develop proficiency with basic addition, subtraction, multiplication and division facts through the use of a variety of strategies and contexts;
- use informal language, mathematical language and symbols to relate problem situations to operations;
- recognize that any one operation can be used to represent diverse problem situations, e.g., subtraction can be used in "take away," as well as comparison, situations;
- construct, use and explain a variety of procedures for performing whole number calculations; and

(continued)

Educational experiences in **Grades 5-8** will assure that students:

- maintain proficiency with basic addition, subtraction, multiplication and division facts through the use of a variety of strategies and contexts;
- develop, use and explain procedures for performing calculations with whole numbers, decimals, fractions and integers;
- understand the concepts of powers and roots, and apply them in problem situations;
- select and use an appropriate method for computing from among mental math, estimation, paper-and-pencil and calculator methods; and
- use relationships among operations and properties of operations (associative, commutative and distributive) as well as order of operations and inverses to simplify computations.

Educational experiences in **Grades 9-12** will assure that students:

- use arithmetic operations to solve problems encountered in everyday consumer situations;
- apply and explain procedures for performing calculations with whole numbers, decimals, fractions and integers;
- use appropriate methods for computing, including mental math, estimation, paper-and-pencil and calculator methods;
- use field properties and the relationship between operations and their inverses to justify mathematical procedures; and
- use absolute value, powers and roots; explore and use negative exponents on integers.

K-12 PERFORMANCE STANDARDS, continued

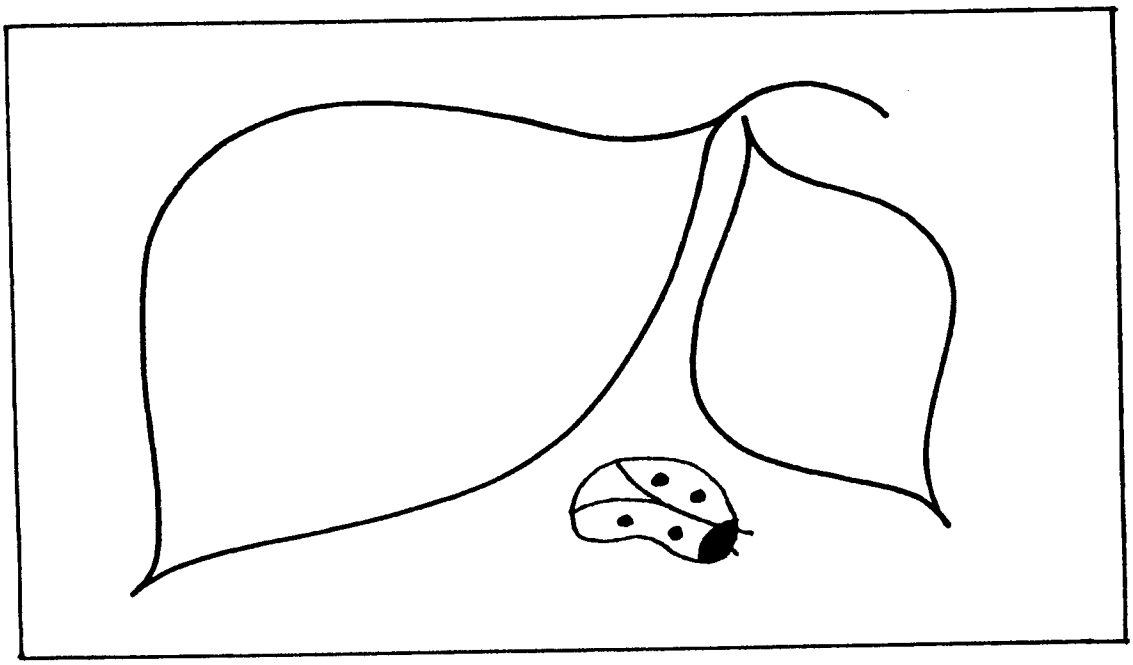
Educational experiences in **Grades K-4** will assure that students: • understand and use relationships among operations, e.g., multiplication is repetitive addition; multiplication is the opposite of division.		

ILLUSTRATIVE TASKS AND EXPERIENCES

As part of ongoing mathematics instruction in Grades K-4, students should have instructional experiences like the following:

1. Ladybugs And Leaves

Create a work mat for each student by drawing two leaf shapes on a piece of 9-by-12 oaktag. Each child will need 10 lima beans (18 if facts through 18 are being explored) to investigate the different ways a number can be expressed as the sum of two addends. For greater motivation, paint (with the help of parents) the beans to resemble ladybugs. Students may, of course, use red cubes or blocks in place of the lima beans.



Each child uses his or her work mat and materials to act out the story read or told by the teacher. Students should be encouraged to create and act out their own stories within their groups.

For a story that calls for six ladybugs, children should be encouraged to find different arrangements on the two leaves. Talk with the class or group about organizing number sentences in some way. Let students make charts for recording their independent findings.

Let students use their materials to answer questions such as:

- How many arrangements are there for five?
- How many arrangements are there for eight?
- Can you predict how many there will be for nine? For 22?
- How did you figure that out?

Use ideas generated by trade books to create additional workmats and painted figures.

Extension: Use base-10 materials and place value mats with hundreds, tens and ones for older students to join or separate sets of multidigit numbers or to partition numbers into three addends.

2. Things That Come In Pairs

Create story situations for students like the following: "In gathering my friends together for a Chinese dinner, I picked up a pair of chopsticks for each of us." Ask questions like:

- How many chopsticks do I alone need?
- How many do my three friends need?
- If everyone in our class eats together with chopsticks, how many will we need?
- Discuss and solve this problem in your group. Write about your solution.
- How many chopsticks for six, 12 or 28 people?

(Students should record their methods and the mathematical notations that represent their solutions.)

Extension 1: Develop a list of things that come in twos. Ask what lists things like tricycles, eggs in a dozen, or innings in a baseball game go on. Extend your lists and create additional lists.

Extension 2: Create a story with the reverse situation: "As we entered the room we were surrounded by cats. Suddenly the lights went out and the meowing started. We could see 28 eyes staring at us in the darkness." Ask questions like:

- How many cats were in the room?
- How many eyes would we see if six cats ran out of the room? (Continue with additional situations.)

3. House Of Operations

Use the following activity with students.

Materials: number squares 1– 18, symbol squares for =, +, -, paper and pencil to record

Work with a partner. Draw a picture of a house. Choose three number squares, one operation square, and the equal sign square from the set of squares below. Arrange these five squares to create a number sentence. Enter your number sentence into the "House of Operations." Create as many true number sentences as you can.

NUMBER AND SYMBOL SQUARES

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	+	-	=

Is the last equation you made true?_____ How do you know?

What are some things you notice about these groups of equations?

Do you see any patterns? Describe them.

Do you think you found all the equations possible?_____ Why do you say that?

Extension 1:
What if your teacher told you that there are 8 possible equations for each set of three numbers? Can you find any equations that are missing from your set?

Extension 2:
Modify the activity to include multiplication number sentences.

4. The Refreshment Stand

Ask the class or students in small groups to devise a menu for a refreshment stand that might sell hot dogs, hamburgers, soda, chips, etc. Prices should be as realistic as possible, but fit the computational skills of the students. Use menu(s) posted on large sheets of newsprint to complete tasks such as the following:

- Make an order and calculate the cost, the change and the composition, e.g., various coins of the change.
- Find orders that come closest to $2 or $5.
- Calculate the cost of snacks for a family of four.
- Ask students to create their own refreshment stand menu problems and share them with classmates.

As part of ongoing mathematics instruction in Grades 5-8, students should have instructional experiences like the following:

1. Catalog Shopping

Allow students to select a popular catalog from the class catalog collection. Tell students they each have won a $300 gift certificate for a catalog shopping spree and can purchase one item for each member of their family. Inform students to use the order form and charts for sizes and shipping/handling charges and to keep track of what they spend so they don't exceed $300. Remind students that the gift certificate will be valid only if they purchase at least three items from their catalogs. Tell students that you will await their order forms and hope they have fun shopping!

2. The Fabulous P B and J Sandwich Sale

Tell students that their task is to determine how much profit is reasonable on each peanut butter and jelly sandwich that they will make and sell at the school fair. First ask students to estimate what they believe is a reasonable profit. Share the following data (or get real data from newspaper flyers or stores):

- A loaf of bread (not counting the heels) provides 20 slices for $1.29.
- A large jar of peanut butter contains 40 ounces or 1.13 kilograms for $5.25.
- A jar of grape jelly contains 22 ounces or 624 grams and sells for $1.89.
- Each sandwich requires two slices of bread, about an ounce of peanut butter and about 3/4 ounce of jelly.

Ask students to use the data to estimate a reasonable profit if they sell 200 sandwiches and to complete the following:

If we sell the sandwiches for $_____ each, we expect to make a profit of $_____. We think this is a reasonable cost per sandwich because.....

3. Anniversary Celebration

Explain to students that they need to help the Lincoln Middle School plan a 35th anniversary celebration. Lincoln consists of:

- ten 6th grade homerooms with an average of 25 students each;
- nine 7th grade homerooms with an average of 30 students each;
- ten 8th grade homerooms with an average of 20 students each; and
- a staff of 60 teachers, custodians, secretaries and other personnel.

The school has a $2,000 budget for the celebration and the planning committee is hoping to provide pizza and soda for all of the participants. The committee also has gathered the following data:

- a large cheese pizza with eight slices costs $10;
- a small cheese pizza with eight slices costs $7.50;
- a two-liter bottle of soda costs 99 cents; and
- a six-pack of soda costs $1.25.

Explain to the committee in writing how you would determine the amount of pizza and soda they would need and determine the cost of these items for the celebration. Will $2,000 cover the costs?

Should they hold a fund-raiser or charge admission? Show how you arrived at your recommendations.

Extension: Plan a celebration for your own school, including menu, purchases and costs.

4. Ski Pass

In groups, let students wrestle with problems like the following and then report to the whole class on their assumptions, reasoning and solutions.

Three family members ski together on weekends. A season pass costs $385 for the first family member, $350 for the second family member, $205 for the third family member, and $125 for a fourth family member. A day pass costs $21.

Explain how you would decide which is the better deal, the day pass or the season pass. Please show your reasoning and present your findings.

As part of ongoing mathematics instruction in Grades 9-12, students should have instructional experiences like the following:

1. The Pizza Order Bid

Provide students with a copy of the takeout menu for one or more local pizza parlors, including costs of different sizes of pizza, costs of additional ingredients, and any coupons or special deals. Ask students to use the menu data to submit a bid for supplying a high school class with enough pizza for a class party. The class estimates that it will need about 100 slices of pizza – half cheese and half pepperoni. Each bid must include the final price (no sales tax because it is a school) and a clear justification of how this price was arrived at. Discuss how to arrive at the lowest bid.

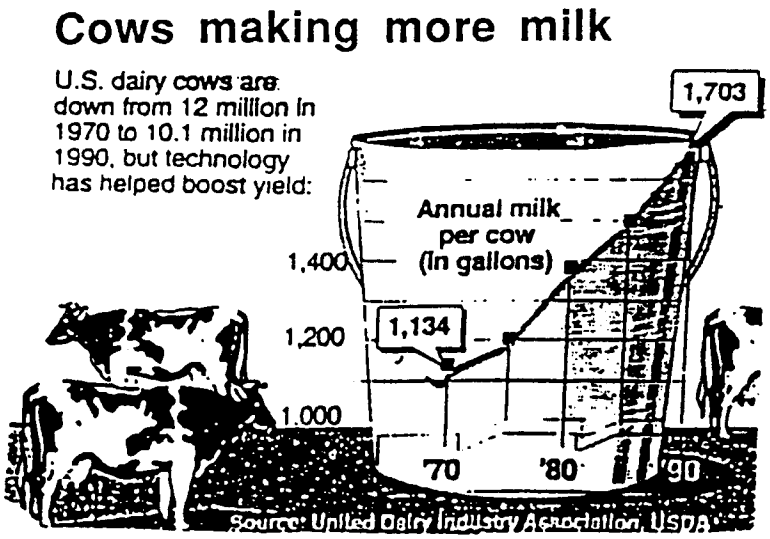
2. Combinations

Remind students that there are 10 digits, 26 letters in the English alphabet, and 52 cards in a deck of playing cards. Use this information to have students explore the calculation of such things as:

- the number of possible license plate combinations when there are two letters and four digits vs. three letters and three digits;
- the number of possible three-digit area codes, with the old restrictions of 2 to 9 for the first digit; 0, 1 or 2 for the second digit and 1-9 for the last digit, vs. the new restrictions of 2-9 for the first digit and any digit for the second or third digit, except no area code can end in 00 or 11;
- the total number of five-card poker hands possible in a deck of cards; and
- a good way to use digits and letters to create a code, assuming one needs a total of 10,000 separate codes for the merchandise in a catalog.

3. Milk, Cows And Your Taxes

Distribute copies of the Milk, Cows And Your Taxes graph and task to students. Encourage students to work on the problems in groups and then share their findings and arguments.



In urging Congress to increase dairy subsidies, the United Dairy Association reports that the number of dairy cows in the United States has decreased from 12 million in 1970 to 10.1 million in 1990. They urge that without subsidies, the number of dairy cows will continue to decrease, threatening the availability of milk by the year 2000.

However, the United States Department of Agriculture reports that the annual average of milk production per cow has increased from 1,134 gallons in 1970 to 1703 gallons in 1990. They argue that increased subsidies are unnecessary because food and technological improvements will continue to boost the average quantity of milk from each cow.

You also know that the population of the United States increased from 200 million in 1970 to 250 million in 1990.

As a consumer of milk and milk products, and as a taxpayer who would have to pay for any increase in dairy subsidies, use the data given to construct a convincing argument for not increasing the dairy subsidies (i.e., "I believe that it is unnecessary to increase dairy subsidies because..."). Make sure to show all of your mathematical calculations.

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USA Today, Port Washington, NY.

4. Fuel Efficiency

Distribute copies of the Fuel Efficiency problem to students. Ask students to work on the problem individually or in groups and then to share their findings and arguments.

FUEL EFFICIENCY

Marissa and Michael both had their own cars. Because they paid all of their fuel expenses, they kept careful records on how much gas they used last month.

Marissa			Michael		
Date	Odometer Reading	Gallons Pumped	Date	Odometer Reading	Gallons Pumped
3/15	8754	7.8	3/18	27537	8.5
3/20	8984	8.4	3/25	27854	9.1
3/28	9249	9.1	4/2	28205	10.3
4/3	9420	7.4	4/9	28457	8.7

Whose car got the better mileage? Explain your reasoning.

On a recent visit to the service station, the mechanic explained to Marissa that underinflation of tires can waste up to 5% of a car's fuel and by simply putting more air in the tires she could substantially boost her car's fuel efficiency. He also reminded her it was time for a tune-up, and a well-tuned car uses up to 9% less gasoline than a poorly tuned car. If Marissa were to inflate her tires and get a tune-up, how many more miles per gallon could she expect?

The amount of carbon dioxide (CO_2) a car emits is directly related to the amount of gas it uses. Cars give off 20 pounds of carbon dioxide for every gallon of gas consumed. How many pounds of CO_2 did Michael's car emit per mile?

PROTOTYPE ASSESSMENTS AND SAMPLES OF STUDENT WORK

As a result of an instructional program in mathematics like that described in this guide, by the end of Grade 4, all students should be expected to complete work like the sample below.

FIRST DAY OF SCHOOL

It's the first day of school and you have been given a $5 bill to buy all of your needed school supplies from the school store. The store has the following supplies:

ITEM	PRICE
Pencil	$0.25
Pen	$0.40
Notebook	$1.25
Crayons	$0.75
Markers	$1.00
Erasers (2-pack)	$0.50
Pencil Sharpener	$0.90
Ruler	$1.00

1. List the things you would buy. Briefly tell why you chose these items.

2. How much would these items cost? Show all of your work.

3. How much change would you get back from your $5 bill? Show all of your work.

Your work will be evaluated on how well you have:

- explained why you bought what you did;
- calculated the total cost of the items you decided to buy; and
- calculated the change you should receive.

USE THIS PAPER FOR YOUR EXPLANATION OF: FIRST DAY of SCHOOL

1 I chose these items becau they were the most important. Like I chose the pencil because I need it to write. I need the ruler to measure. I used a calculat to add it up. and it is right on the dot of $5.00

2

0.50
1.25
1.00
0.50
1.00
0.75
5.00

3

$5.00
$5.00
0.00 change

As a result of an instructional program in mathematics like that described in this guide, by the end of Grade 8, all students should be expected to complete work like the sample below:

SHOWING SCHOOL SPIRIT

You and your four friends are going to make signs for your school pep rally. Since you're all pitching in your allowance money, you need to decide where to buy the materials at the cheapest price. You need a roll of paper, markers, poster paint and paint brushes. The largest size poster your school allows is 3 feet by 7 feet, and you and your friends want to make your posters as big as possible so they are sure to be seen by everyone! Standard rolls of paper are 3 feet (1 yard) wide. Check prices by reviewing the advertisements shown below.

1. Where will you go to buy specific supplies if all of the stores are within walking distance?
2. If you need to drive to each of the stores, and your mom will only take you to one store, which one would you choose? Why?

To successfully complete this task you are expected to:

- decide how many signs you'll make and how much of each supply you'll need;
- compare the cost of each item needed by calculating the price per unit; and
- show your work to support your decisions.

Your work will be evaluated on how well you have:

- accurately calculated each price per unit;
- chosen the best place to shop for each individual item;
- chosen the best place to shop for all items at one store; and
- defended your purchasing decisions.

MARI'S MURAL MARKET	
7 Yards of Paper	$2.00
10 Big Bold Markers	$2.50
7 oz. Poster Paint Comes in 4 Colors	$1.75 per color
5-Pack, Large Paint Brushes	$3.50

STEVE'S SIGN STORE	
15 Yards of Paper	$4.00
8 Colorful Markers	$2.00
12 oz. Poster Paint Comes in 8 Colors	$3.00 per color
1 Big Paint Brush	$0.75

AL'S ART SUPPLIES	
10 Yards of Paper	$3.00
6 Thick Bright Markers	$1.50
8 oz. Poster Paint – Variety of colors	$1.85 per color
3-pack of Big Brushes	$2.00

For this math P.O.W., we have to make signs for the school pep rally. We have to decide where to buy the materials at the cheepest prices. We need a roll of paper, markers, poster paint and paint brushes. The largest sized poaster you can have is 3ft by 7ft. Here are the stores we came up with:

STEVE'S SIGN STORE	
15 Yards of Paper	$4.00
8 Colorful Markers	$2.00
12 oz. Poster Paint Comes in 8 Colors!	$3.00 per color
1 Big Paint Brush	$0.75

MARI'S MURAL MARKET	
7 Yards of Paper	$2.00
10 Big Bold Markers	$2.50
7 oz. Poster Paint Comes in 4 Colors!	$1.75 per color
5 Pack Large Paint Brushes	$3.50

AL'S ART SUPPLIES	
10 YARDS OF PAPER	$3.00
6 THICK BRIGHT MARKERS	$1.50
8 OZ. POSTER PAINT - VARIETY OF COLORS!	$1.85 PER COLOR
3 PACK OF BIG BRUSHES	$2.00

If all of the stores we in walking distence we have to find out which one we would go to. Also, if you need to drive to each of these stores and we can only go to one, which one would we go to and why. We have to accurately calculate each price per unit & Choose the best place for each individual item & and choose the best place to shop for all items in one store. We have to defend our purchasing decisions.

In order to plan & to solve the P.O.W., we divided the work up into the sections. Since you had to figure out the unit price of

each item We did this first so we could figure out the answers to the problems. When we first looked at this problem, we were a little confused on how to go about solving this. But as time went by we worked on this and figured the solutions to this problem. We predicted that Mari's Rural Market would be the best place to shop. We thought this because the prices seemed to be the cheepest. But as we did the unit pricing our prediction was wrong ——— ↘ over

Work/Answers

Mari's Mural Market

$\frac{\$2.00}{7 \text{ yds}} = .28 = 30¢$ per yard

~~.29~~

$\frac{\$2.50}{10 \text{ markers}} = .25$ per marker ← doesn't matter

$\frac{\$1.75}{7 \text{ paints}} = .25$ per color

$\frac{\$3.50}{5 \text{ brushes}} = .70¢$ per brush

Steve's Sign Store

$\frac{\$4.00}{15 \text{ yds}} = 26.\overline{6} = .27$ per yard ← best

$\frac{\$2.00}{8 \text{ markers}} = .25$ per marker

$\frac{\$3.00}{12 \text{ oz}} = .25$ per oz

$\frac{\$0.75}{1 \text{ paintbrush}} = .75$ per brush

AL's ART SUPPLIES

$\frac{\$3.00}{10 \text{ yd of paper}} = .30$ per yard

$\frac{\$1.50}{6 \text{ markers}} = .25$ per marker

$\frac{\$1.85}{8 \text{ oz of poster paint}} = .23$ per oz. ~~paint~~ - best

$\frac{\$2.00}{3 \text{ packs of brushes}} = .67$ per pack - best

Explanation of work:

What worked to find unit price was taking the dollar amount and dividing it by the amount of products, to get the unit price.

Then look at all the unit prices for all 3 stores and see which is the best to buy all your product or to buy the products seperately.

For the best store all together it would be Al's Art Supply. One →

Answer

Item is a couple cents extra/more but everything else is cheeper. You want to go to the cheeper store because it is your money and you want to spend it wisely. For the yards of paper the best is .27 per yard at Steve's Sign Store. For the markers it doesnt matter where you go because it is .25 at all three place. The best buy for the poster paint is .23 per oz at Al's Art Supply and the brush are the best buy there for .67 per pack. The size signs we are making is ~~3~~ three 5 yd x 3ft signs. We will use 3 paint brushes, 3 12oz. paints and 8 markers.