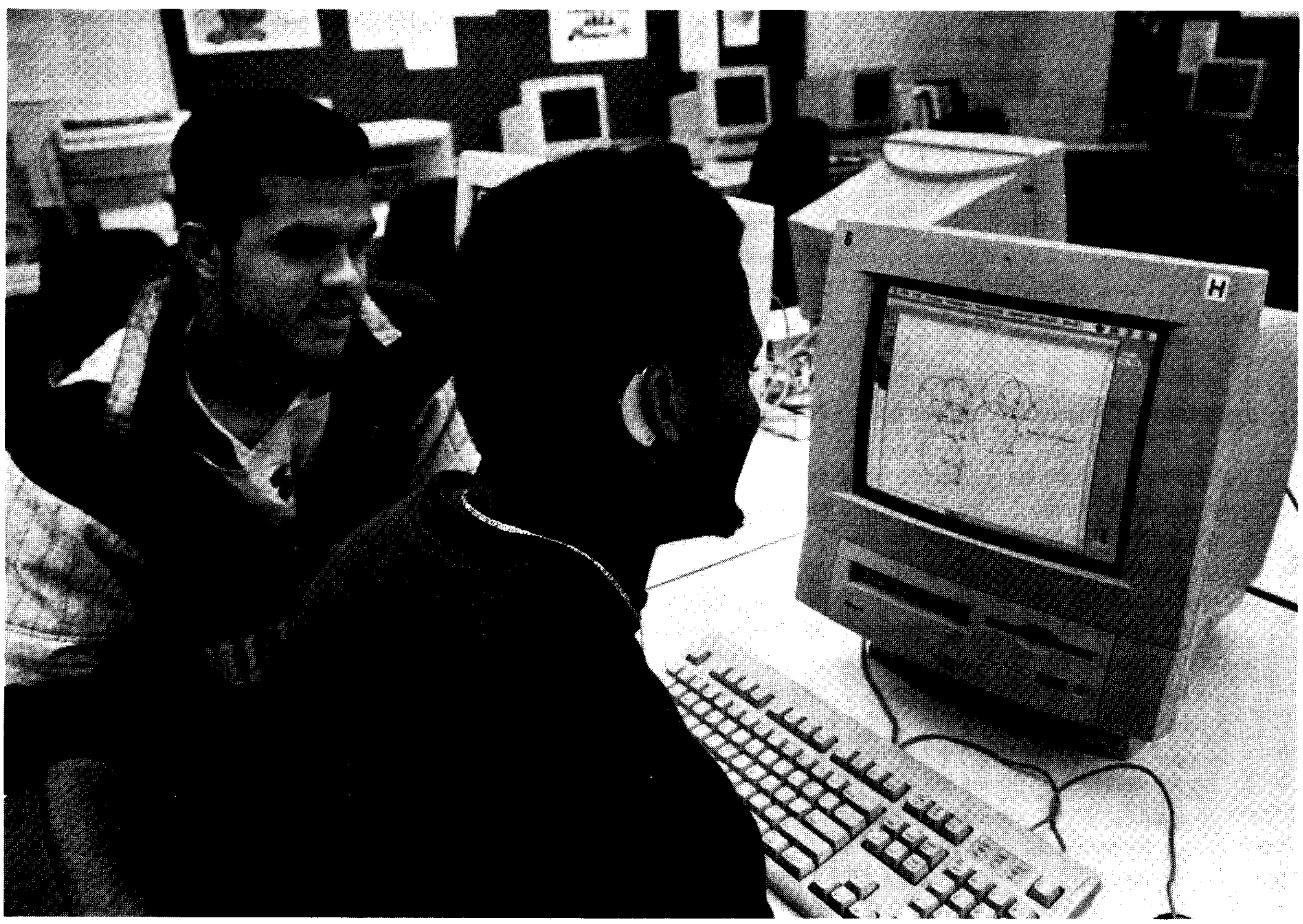
Critical Issues In Mathematics Programs 5

Reading And Writing In Mathematics
Integrating Mathematics Across Disciplines
Customary And Metric Measurement
Grouping, Tracking And Inclusion
Manipulative Materials
Calculators And Computers
Alternative Scheduling
Equity And Diversity
Parent Involvement And Support
Recognizing A Good Math Classroom



As educators use the content delineated in Chapter 2, the program components described in Chapter 3, and the development process outlined in Chapter 4, they will face several additional issues that tend to be unique to mathematics. Each of these issues is complex and comes with few easy answers. Each must be faced head-on if we are to truly make mathematics work for all.

Reading And Writing In Mathematics

It used to be that the "three R's" were separate and distinct. It was thought that a lot of mathematics – meaning arithmetic – could be learned regardless of a student's reading and writing abilities. However, as applications and problem solving have become more significant parts of the mathematics program, and as the language arts have been more broadly integrated across the curriculum, reading and writing have assumed an increasing role in high-quality mathematics programs.

Reading skills become more and more important as teachers reduce the emphasis on pages of wordless, rote computation and symbol manipulation and increase the emphasis on problems, applications, and the use of contexts like those illustrated in Chapter 2. Similarly, writing skills become more and more important as teachers use projects, open-ended tasks and other written means to develop, as well as assess, mathematical understanding and communication.

In fact, the shift toward making mathematics instruction as language-rich as possible reinforces language arts skills as well as improves the quality of the mathematics program. Rather than making it more difficult to learn mathematics, language-rich programs have been shown to be more effective for all students, particularly for those with limited English proficiency.

Integrating Mathematics Across Disciplines

Just as the mathematics program is a necessary and natural place to reinforce language arts skills, the science, social studies, health education, arts, home economics and technology education programs are ideal places into which mathematics can be integrated, and from which rich and relevant contexts for applying mathematical skills and concepts can be drawn.

Mathematics, therefore, should have a visible place in the curriculums of other disciplines and other disciplines should be clearly visible within the contexts, tasks, projects and activities of the mathematics program. Whether linking art to geometry, statistics to a science experiment, or technology education to measurement, these cross-disciplinary connections must be carefully built into the curriculum development process and can significantly enhance the relevance of learning for students.

Customary And Metric Measurement

The ongoing debate over when or whether the United States will "go metric" leaves the mathematics curriculum developer in the somewhat awkward position of having to **do both.** Any observer of the current state of affairs and the world of the foreseeable future will recognize that our students must be prepared to be "bilingual" in terms of measurement. That is, the world will continue to bombard Americans with quarts of milk, two-acre lots, two-by-fours for studs and tons of trash – all requiring a thorough familiarity with the customary system of length, weight and capacity. However, Americans are increasingly confronted with 25-milligram doses, 2-liter bottles and 1500-meter races that all argue for an equally thorough familiarity with the metric system of length, mass and volume. Like the Connecticut Mastery Test (CMT), the Connecticut Academic Performance Test (CAPT), and most recently produced instructional materials, an effective program provides a balanced approach to measurement units – one part customary and one part metric.

Perhaps more important than the issue of customary vs. metric, however, is the broader issue of the importance of measurement generally. The study of measurement – customary and metric – is frequently omitted from the implemented curriculum, and performance on some CMT and CAPT items reveals serious gaps in student understanding of basic measurement. Therefore, the key issue is assuring adequate attention to the concepts of measurement through the use of nonstandard measures (e.g., pencil lengths or paper clip weights), and equally strong attention to **both** systems of measurement units. In fact, a curriculum based on problem solving is one that regularly asks questions like the following: How far? How much? How big? How much bigger? These questions and others like them all have answers with units like minutes, dollars, ounces, pounds, milligrams or meters, as well as square feet and kilometers per hour. These kinds of problems should encourage all to increase the curricular emphasis on and the instructional time allotted to measurement.

Grouping, Tracking And Inclusion

What was written in Chapter 3 bears repeating: "No single component of the educational system more powerfully communicates the expectations – both high and low – we hold for young people than the ways in which schools group, sort and track students. A major step in moving toward the vision of "mathematics for ALL" is a dramatic decrease in the ability-grouping and tracking of students. This does not mean abandoning all ability groups; it does not mean the elimination of all honors courses; and it does not mean all students grouped heterogeneously all of the time. It does mean, however, a change in policy at the school and district levels regarding ability-grouping and tracking so that no student is denied access to a rich and demanding mathematics program best suited to his or her individual needs and interests."

The grouping and tracking of students was an acceptable and inevitable strategy for implementing a skill-based mathematics program, assessed with norm-referenced tests and designed primarily to sort out students for a world of work that was based on many assembly line workers and few managers. As technology has diminished the need for mastery of complex paper-and-pencil skills, and as the world of work has demanded broader problem-solving skills on the part of nearly all students, the rationale for tracking and grouping has started to weaken. The increasing call for inclusion and greater heterogeneity must be seen as a call for greater opportunity for all. It is also a call for changes in instructional patterns so that students are much more actively involved in their learning.

Regardless of grouping policies, most teachers are well aware of the broad range of abilities, styles, interests, motivations and rates of learning in every class, and these teachers recognize that they must provide for the individual needs of **all** students, be they high, low or in-between. The fact that more heterogeneous classes, often including special education students, usually are more difficult to teach must be balanced with the understanding that research shows clearly that **tracking**:

- increases the gap of opportunity to learn;
- results in unfair and disproportionate placement of poor and minority students in lower tracks;
- inappropriately isolates students primarily on the basis of how fast they learn;
- results in lower track students never catching up; and
- lowers self-esteem and increases behavior problems of lower-track students.

These outcomes, even if the result of the best of intentions, are no longer compatible with the needs of today's workplace or the goals of our democracy.

Manipulative Materials

Manipulatives – concrete physical objects that can be viewed, handled, arranged and taken apart to illustrate and exemplify concepts – are highly useful tools in mathematics classrooms at all levels. Research demonstrates clearly that most people learn better with hands-on experiences. Manipulatives are powerful tools for these hands-on experiences and serve as a natural bridge from concrete to abstract understanding of mathematical ideas. In addition, these materials appeal to a variety of senses and, thereby, provide greater access to mathematical ideas.

As is the case with all tools, the use of manipulatives is not nearly as important as **how** they are used. Such critical understandings as the concept of number, the meaning of place value, the concept of fraction, the "why" of the division algorithm, the meaning of "mean," or visualizing what happens when an area is rotated into a volume in calculus, all can be more effectively taught through the use of manipulative materials. It is important to remember that students cannot learn mathematics well with manipulatives alone. However, instruction that makes effective use of manipulatives is likely to be far more effective than instruction based solely on textbooks and chalkboards. (Appendix E provides cross-walks between core concepts and key manipulative materials.)

Calculators And Computers

What the pencil was to yesterday's mathematics and writing, the calculator and the word processor are to today's mathematics and writing: critical tools that, when used appropriately, enhance learning.

The heart of the debate over the appropriate use of technology in mathematics classes is whether these tools are labor-saving crutches that discourage the development of key skills and discipline **or** powerful instructional tools that strengthen problem-solving skills, provide increased access to mathematical ideas and prepare students for the world of work, where technology is ubiquitous. The underlying philosophy of this guide, as well as that of the National Council of Teachers of Mathematics, is the latter – that calculators and computers are essential tools for teaching mathematics and should permeate a high-quality program. Obviously, as with all tools, there is a time and a place for their use. An effective program recognizes that, while students are learning to master basic number facts and to develop estimation skills, calculator use should be very limited.

While the power of calculators is constantly changing, it is recommended that primary students have access, when appropriate, to simple four-function calculators to solve problems, develop place value understandings, and strengthen their command of the four basic operations. Intermediate students should have access to fraction-capable calculators that, in addition to what four-function devices can do, allow students to strengthen fraction and decimal number sense. From middle school through high school it is recommended that some form of graphics calculator with a multiline screen, graphing and tabular representations, and all the features of a scientific calculator be available to strengthen prealgebraic, algebraic and statistical understanding. In fact, the illustrative tasks described in Chapter 2 assume the foregoing calculator recommendations.

As calculators become more and more powerful, the line between what is a calculator and what is a computer becomes more and more blurred. For example, many are calling the TI-92, with a QWERTY keyboard, a "hand-held computer," since it is capable of symbol manipulation, extensive data manipulation and dynamical geometry. However, until such hand-held computers are on every student's desk, the classroom computer can be used for demonstration and for independent work.

Alternative Scheduling

At the time of this writing, another controversial issue – particularly at the high school level – is alternative scheduling schemes, including the block schedule. For many, block scheduling is an idea whose time has come, and changing the core schedule by which high schools are organized is a long overdue step toward revitalizing these schools. From the perspective of the mathematics program, proponents argue that block schedules offer several powerful improvements for teachers and students.

First, it is widely recognized that a hands-on, activity-based laboratory approach, complete with projects, tasks, experiments and technology, requires more time than the traditional 45- or 50-minute class period. Block schedules that provide 80- and 90-minute periods are more conducive to the kinds of instructional shifts being advocated. Second, it is widely recognized that a daily load of 100 to 140 students is much too large if one is going to assign additional tasks and provide meaningful feedback on student work. Schedules that reduce student course loads to three or four courses per semester also reduce teacher loads to three courses and approximately 75 students, creating more reasonable opportunities for larger-scale projects and longer-term activities.

A major drawback that teachers of mathematics face within such block schedules is the possibility that students may go nearly a year without a mathematics course. Proponents of block scheduling say that this is rare, due to the fact that students are taking more mathematics and most students follow a consistent pattern of mathematics each first semester or each second semester. Others suggest that there is little difference between a semester of geometry in a block schedule and a full year of geometry in a traditional schedule, and that concerns about the need for daily exposure to mathematics are unfounded.

Equity And Diversity

Equity and diversity are broad goals of every school district. However, like all broad goals, they must be translated into daily classroom practice. Such equitable classrooms are easy to recognize.

- All students, girls as well as boys, African-American and Hispanic as well as white, shy as well as assertive, quiet as well as loud, speak up in class regularly.
- No student is allowed to "put down" or pick on another student.
- High expectations are communicated clearly to all students, not just those who appear to "get it."
- Praise is based on achievement, not neatness of work, compliance or appearance.
- All students experience leadership roles.
- All students have an opportunity to learn the material presented.
- The contributions of female and male mathematicians and scientists from different ethnic groups are embedded within the curriculum.
- Stereotypes about who does and who does not do mathematics are confronted directly.

These characteristics are key pieces of a program that truly makes mathematics work for all and, therefore, must be components of a professional development program, as well as of the professional supervision and evaluation processes. A more detailed "Profile of an Equitable Math and Science Classroom and Teacher" can be found in Appendix G.

Parent Involvement And Support

It is increasingly clear that no real reform of mathematics programs is likely until there is parental support for the changes being proposed. Parents are legitimately wary that reform is just a new version of the "new mathematics" of the past. They are legitimately concerned that educators once again are experimenting with their children. And they are legitimately worried that this reform will be just another passing fad that leaves their children unable to compete in the workplace.

These concerns can only be addressed through ongoing communication and the development of trusted relationships between parents and schools. This communication can be fostered by inviting parents into classrooms, encouraging parents to volunteer during mathematics classes, and being available to answer parental questions and concerns. This communication also occurs through annual "math nights" conducted at schools, periodic letters home explaining the mathematics program in which students will be engaged, and conferences that allow parents to express their concerns and view the results of the program.

In addition, parental support can be garnered by encouraging parents to make mathematics commonplace at home. Parents can help their children and support the mathematics program by making the mathematics of everyday life a part of the interaction with their children. This can be done in fast-food restaurants and with menus when students estimate costs and tips; at the gas pump where decimals and rates are clearly displayed; at the bank with the deposits and withdrawals; with cooking recipes which include fractions and measures; at sports events and with the newspaper sports section filled with statistics; or at the grocery store with costs and unit costs. This "conversational math" that helps build number sense and estimation can be as important as reading to children.

10 Expectations Parents Should Have About Their Children's K-8 Mathematics Program

In forging a positive relationship with parents, it is important that parents have high and clear expectations for their children's achievement. The following "10 Expectations Parents Should Have About Their Children's K-8 Mathematics Program" were developed by the Connecticut Academy for Education in Mathematics, Science and Technology.

1. Parents should expect that the way mathematics is being taught to their children will be very different from the way they were taught 20 or 30 years ago.
2. Parents should expect that their children will be using calculators regularly.
3. Parents should expect to see their children doing few repetitive and tedious drills, such as multiplication tables and long division.
4. Parents should expect that their children's mathematics classes and homework will include solving interesting and relevant mathematics problems, gathering and analyzing data, justifying solutions and writing conclusions.
5. Parents should expect that the mathematics their children are learning will be beneficial and applicable to life outside of school.
6. Parents should expect that their children will be prepared and encouraged to take algebra and geometry during their high school years.
7. Parents should expect that their children's mathematics achievement will be assessed and reported on the basis of their problem-solving abilities, projects and portfolios of work (done individually or as part of a group), not on the basis of mastery tests and standardized tests alone.
8. Parents should expect that mathematics will be enjoyable for their children and that mathematics classes, activities and assignments include hands-on experiences that are likely to excite and encourage their youngsters.
9. Parents should expect that, if or when these expectations are not being met, they (and their questions) will be welcomed by school personnel; and that parents will be valued for caring enough to ask questions.
10. Finally, parents should expect that learning takes work and discipline.

Recognizing A Good Math Classroom

The Mathematical Sciences Education Board, in 1997, developed two sets of indicators – what both students and teachers would be doing – in order to recognize "a good math classroom when I see it." These indicators, published in a brochure *What Should I Look For in a Math Classroom?* are reprinted with permission.

> A math classroom should provide practical experience in mathematical skills that are a bridge to the real world of jobs and adult responsibilities. This means going beyond memorization into a world of reasoning and problem solving. *Sounds good, but how will I recognize a good math classroom when I see it?*

What Are Students Doing?

- ☑ Interacting with each other, as well as working independently, just as adults do.
- ☑ Using textbooks as one of *many* resources. Students should know *how* and *when* to use manipulatives (such as blocks and scales) and technology (such as calculators and computers) as problem-solving tools.
- ☑ Applying math to real-life problems and not just practicing a collection of isolated skills. Lots of time is allowed for solving complex problems.
- ☑ Seeking a best solution among several solutions to a problem. Students can explain the different ways they reach these solutions, and defend the choice of one over another.
- ☑ Working in groups to test solutions to problems with each group member highly involved.
- ☑ Communicating mathematical ideas to one another through examples, demonstrations, models, drawings and logical arguments.
- ☑ Working in teams to challenge and defend possible solutions. Students help each other to learn.

What Are Teachers Doing?

- ☑ Guiding students in exploring multiple solutions to any problem; challenging students to think deeply.
- ☑ Moving around the room to keep everyone engaged in productive work.
- ☑ Encouraging students to raise and discuss questions about math for which there are no textbook answers. Rather than simply answering these questions, teachers are helping students to gain mathematical competence and confidence by finding their own solutions.
- ☑ Guiding students in making appropriate use of manipulatives and technology.
- ☑ Promoting student use of inquiry and creativity. Students are moved to higher levels of learning by pursuing alternative approaches to solving a problem or by proposing new problems that are variations on, or extensions of, a given problem.
- ☑ Bringing a variety of learning resources, including guest presenters, into the classroom in order to increase learning options for all students.
- ☑ Working with other teachers to make connections between disciplines to show how math is a part of other major subjects that students are studying.
- ☑ Using assessment that focuses on problem solving and understanding rather than on memory and speed.
- ☑ Helping all students to explore career opportunities that use the mathematics that they are learning.

From *What Should I Look For in a Math Classroom?*
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