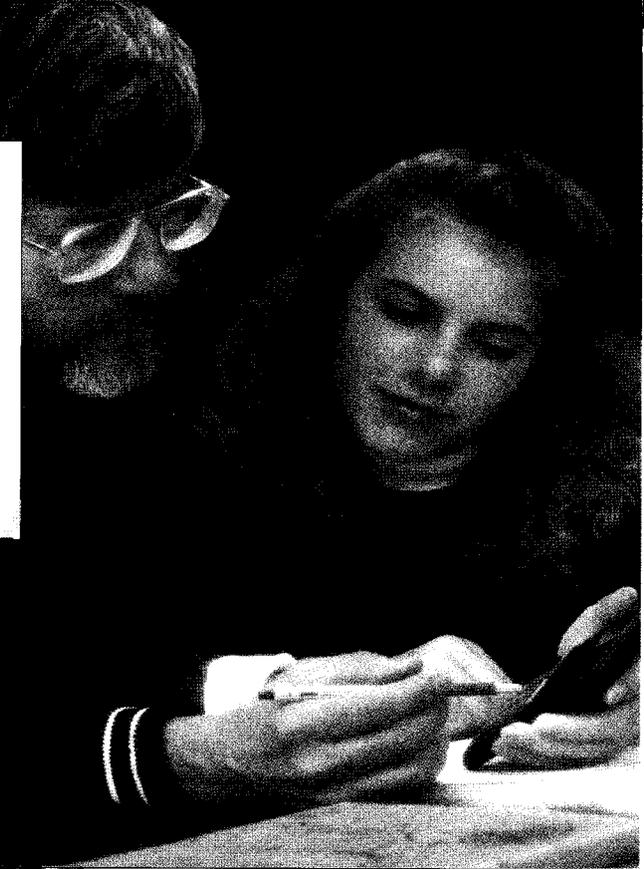
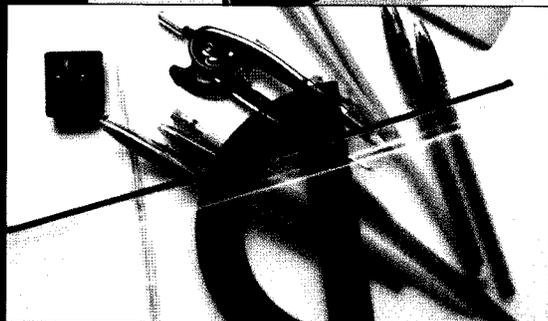


# Mathematics



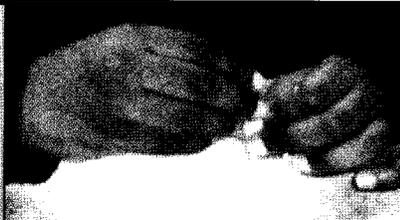
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## A Guide to K-12 Program Development

State of Connecticut  
State Board of Education 1999



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**A GUIDE TO K-12  
PROGRAM DEVELOPMENT  
IN MATHEMATICS**

Photos inside this guide were taken at

- Wethersfield High School and
- Maloney Interdistrict Magnet School in Waterbury.

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## PREFACE

We are proud to provide you with this *Guide to K-12 Program Development in Mathematics* as a resource to further the reform of mathematics education in Connecticut and to continue our state's tradition of high-quality mathematics instruction and relatively high levels of student achievement.

This guide has been designed for educators across the state who are charged with planning, implementing, evaluating and improving kindergarten through Grade 12 mathematics programs. As such, this guide should be viewed and used as a "toolkit" of suggestions, examples and ammunition for:

- building an effective curriculum that should propel students into the 21st century, armed with the mathematical power our society and our economy needs;
- implementing an effective program that empowers all students mathematically; and
- addressing critical issues that face all developers and designers of school mathematics programs.

While this guide can be used in its entirety, each chapter has been designed to stand alone and provide answers to the core program development questions:

- Chapter 1 lays out a case for change and answers the question, "Why bother?"
- Chapter 2 delineates a recommended K-12 curriculum and answers the questions, "What should we teach?" and, "What does the curriculum look like?"
- Chapter 3 describes the components of an effective program and answers the question, "What are the elements of a high-performance mathematics program?"
- Chapter 4 outlines the curriculum development process and answers the question, "How can we create a high-quality curriculum guide?"
- Finally, Chapter 5 addresses common and often controversial issues that are faced while designing and implementing K-12 mathematics programs.

We live in a world that bombards us with data, wherein mathematics provides the tools for making sense of this data. We live in a world of nonstop change, wherein mathematics is how we quantify this change. We live in a world of patterns, wherein mathematics is the language of patterns. This is a world that demands more mathematics and better mathematics for more students. We believe that this guide offers a road map to the type of mathematics program that meets these expectations and demands.

In the final analysis, this guide represents a call for change in what mathematics we value, in how best to teach and assess that mathematics, and in our expectations for who can learn mathematics. Such changes, described throughout the guide, are predicated on a series of shifts in practice and mindsets, a balanced view of the mathematics curriculum, and a nonnegotiable belief that all students can learn mathematics. Each of these changes comes gradually and each can be controversial. Connecticut's students, however, deserve no less than a mathematics program that maximizes both opportunity and achievement. We wish you much success using this guide to design and implement such world class programs.

Steven Leinwand and Mari Muri  
Consultants in Mathematics

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# INTRODUCTION

Meeting our collective goal of maintaining a strong democracy requires a citizenry literate in many fields, including mathematics. School mathematics programs, therefore, play a crucial role in assuring that all students have the opportunity to develop mathematical power and the ability to apply mathematical skills and concepts in diverse situations. All around us are compelling reasons for building powerful school mathematics programs:

- The world of work – particularly Connecticut’s high-tech workplace – increasingly requires mathematical know-how to maintain productivity and economic viability.
- The demands of citizenship – exemplified by such controversial issues as taxes, solid waste disposal, open-space vs. development, integrating schools or reducing crime – increasingly requires mathematical literacy to respond intelligently to these and similar issues and to assure a truly informed electorate.
- The impact of technology – in the form of calculators and computers - has rendered some mathematics less important, while making other mathematics more important than ever, and still other mathematics finally accessible to all.
- Mathematics is part of the culture in which we live and an appreciation of that culture helps enhance the quality of life for all citizens.
- And a growing body of research – particularly in the field of cognitive psychology – broadens significantly our understanding of how students best learn and come to understand mathematics.

In other words, the world that our students are being prepared for is far different from the world for which we – their parents and teachers – were prepared. Our society is changing rapidly and radically, requiring more and better mathematics for more students. The situation is clear:

- We must strengthen students’ conceptual understanding of **when** and **why** we compute, in addition to assuring that students know **how** to compute.
- We must assure that every student leaves school possessing number sense, data sense, spatial sense and symbol sense, because broad mathematical literacy is truly needed by all, no longer just a few.
- We must draw from research and the wisdom of practice to create classrooms where the active construction of mathematical meaning arises from realistic applications, concrete materials and pictorial models, in lieu of classrooms where the primary method of instruction entails little more than the passive memorization of fragmented bits of information.
- And we must recognize that changing conditions require more than just cosmetic changes in Connecticut’s school mathematics programs.

This *Guide to K-12 Program Development in Mathematics* has been developed in response to these issues and to these needs for change. The Framework describes a vision for school mathematics that is based on a set of clear beliefs about teaching and learning, and that is closely aligned with that of the National Council of Teachers of Mathematics. The Framework then fleshes out this vision by suggesting appropriate content, illustrative tasks, and samples of high-quality student performance. Then, to help move from words to concrete classroom reality, the Framework describes necessary program elements of a world-class mathematics program and suggests a series of implementation strategies for making it all happen. More specifically, this Framework is designed to:

- assist Connecticut teachers, administrators and policymakers in implementing instructional programs in mathematics that are responsive to today's needs;
- delineate – by grade level clusters and by sub-discipline strands – key mathematical experiences for Connecticut students and our expectations for what these students should know and be able to do;
- delineate the components of a world-class K-12 mathematics program, including a set of program delivery standards for the effective implementation of this framework in Connecticut schools and school districts; and
- accomplish these goals in a friendly and readable tone that speaks to the diverse audiences of the lay public, district and school administrators, teachers and parents, but more directly to the district-level mathematics infrastructure in each of Connecticut's school districts.

We hope that readers and users of this framework will find it a rich and helpful resource for revising curriculum, shifting instruction, fortifying assessment and strengthening the overall effectiveness of K-12 mathematics programs for Connecticut's students.

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