# ILLUSTRATIVE TASKS AND EXPERIENCES

**As part of ongoing mathematics instruction in Grades K-4, students should have instructional experiences like the following:**

## 1. Farlings And Fuzzles

Students are asked to read the following story: "Dr. Feather is trying to find out how many birds are feeding in the marsh. The first week he counts 5 long-necked farlings and 5 short-beaked fuzzles. The next week he counts 10 farlings and 6 fuzzles. In the third week he counts 14 farlings and 8 fuzzles. In the fourth week he sees 19 farlings and 11 fuzzles. At the end of the fifth week he counts 23 farlings and 15 fuzzles. Dr. Feather thinks there is a pattern to the increasing number of birds." Students are asked to make a table of the number of each type of bird, look for patterns, and see if they can predict the number of farlings and fuzzles Dr. Feather should expect at the end of the sixth and the seventh weeks.

## 2. A Tale Of Two Offers

Students are told that a friend is going on vacation and needs someone to collect her mail and newspapers while she is gone. She is offering to pay in one of two ways.

- Offer #1: She'll pay you $1 for each of the 10 days.
- Offer #2: She'll pay you 10¢ the first day, double that on the second day, double that amount on the third day, and so on for the 10 days.

Ask students which offer they would take. Ask students how much more they would make by choosing this offer. Make sure students explain their reasoning and show their work.

## 3. Patterns On The 100s Chart

Explore the many patterns on the 100s chart (1 to 100 in 10 rows) by engaging students with tasks such as the following.

- What patterns do you see on the chart? Describe these patterns to a partner.
- Place counters on multiples of 3 (3, 6, 9...) and ask what patterns are seen.
- Close your eyes as I move one or more of the counters on the display chart. When you open your eyes, tell me which counter I moved. How do you know?
- Describe the pattern seen with all 5s in the chart covered.
- What numbers come next in the following sequences? What makes you think so?
  8, 14, 20, 26, 32, 44, 50,..............
  9, 13, 17, 21, 25, 29, 33,..............
  27, 41, 55, 69, 83,..........................
- Find and cover 7, 16, 25, 34, 43, 52 and 61 on the chart. Describe the pattern visually, spatially and numerically.
- If I move a counter one space to the right (or one left, or one up, or one down and one to the left, and so on...), what is the new number's relationship to the previous number?

## 4. Quilt Patterns

Read the book *The Patchwork Quilt*, by Valerie Flourney (Dial Books and Doubleday/Dell, 1985), to students and discuss how the quilt patterns are developed. Bring different quilting books into the classroom for students to look at and enjoy. Have students look at the fractional components of the quilt patterns. Have squares of various colors cut to the size of the squares for the class patchwork quilt. Have students do folding and cutting of the squares to create regions of halves, quarters and eighths. Let them make quilt squares by pasting down different fractional regions they have used to create their squares. Students can sort and classify the different patterns created. The final product can be a class quilt put together by patterning the different quilt squares.

## 5. Pattern Block Patterns

Each student is asked to make a pattern that is about 20 blocks (or cubes) long and to draw and color a picture of their patterns. Have students use their patterns to answer questions such as the following.

- How many blocks of each color did you use so far?
- What color will the 25th block be?
- For 30 blocks, how many blocks of each color will there be?
- How many of each shape have you used so far?
- What shape will the 20th pattern block be?

**Extension:** What if you wanted to know what color the 100th block would be? How could you find the answer?

## As part of ongoing mathematics instruction in Grades 5-8, students should have instructional experiences like the following:

## 1. The Amazing Half

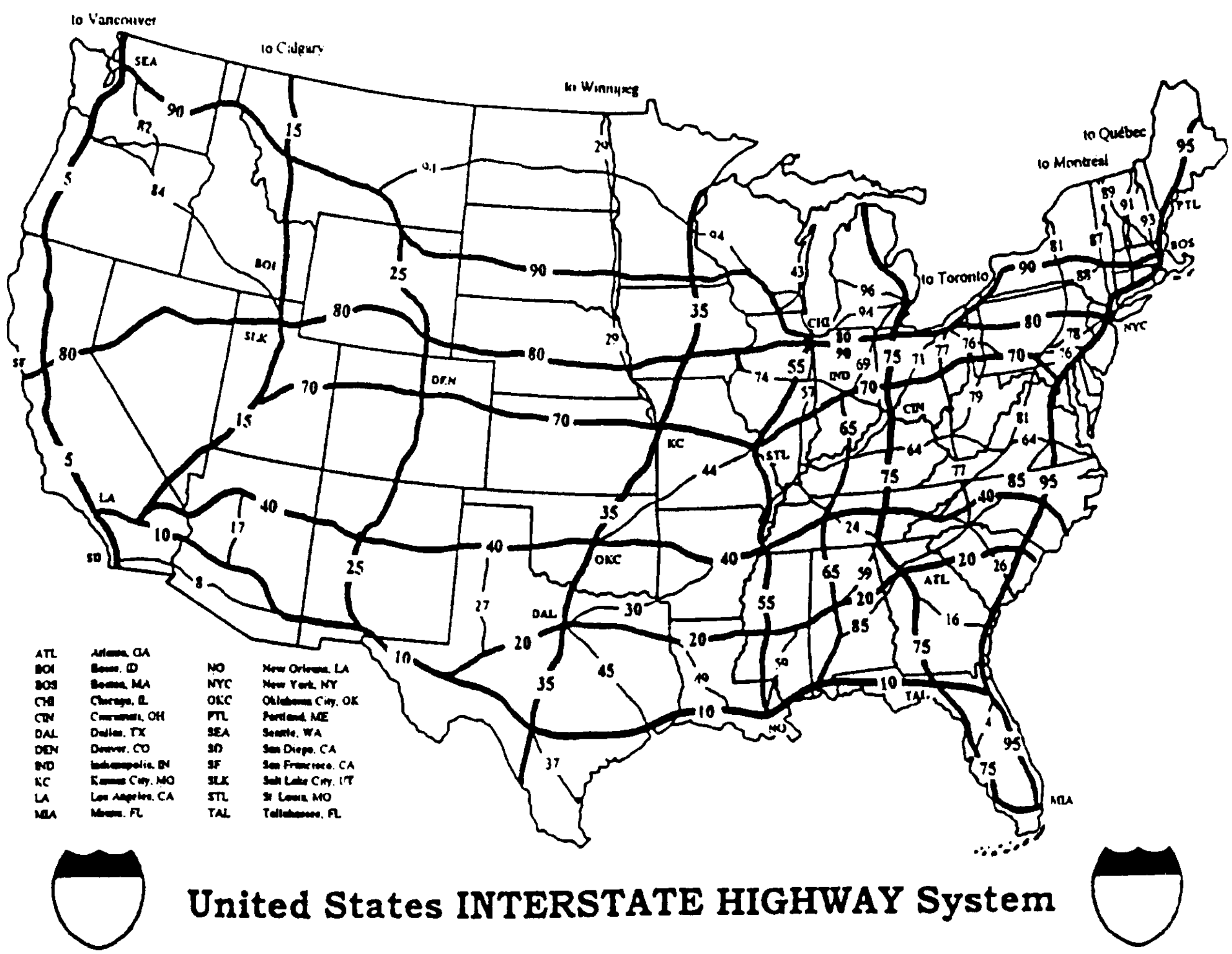
Students are asked to imagine the following situation: One of your friends decides to walk halfway home with you. When that friend stops walking with you, another friend walks half of the rest of the way home with you. What fraction of the total distance will this second friend walk with you? When your friend has walked half of the rest of the way with you, another friend goes with you half of the rest of the way home, and so on. What fraction of the total distance will the third friend walk with you?

Ask students to discuss this situation and draw a picture of it. Then ask students to consider the following: How many friends would you need to continue this pattern all the way home? Will you ever get home? What is so "amazing" about one-half? Finally, ask students to write about this situation in their journals.

## 2. My Way And The Highway

Using a map of the United States similar to the one below, ask students to make conjectures and identify patterns in the numbers assigned to interstate highways that crisscross America. For example, students might:

- consider the major interstates that run vertically and those that run horizontally;
- consider the major interstate highways that connect horizontally and vertically and how their numbering systems relate;
- consider how beltways, or loops around major cities, are numbered and how this system is limited to, and different from, the numbering of "spurs" that lead from interstates into major metropolitan areas;
- explain why I-35 roughly bisects the United States from north to south, and not I-55 as you might expect;
- locate I-43, which connects Milwaukee and Green Bay, Wisconsin (Do you think this interstate has been misnumbered? Explain your reasoning);
- find any other interstates that they think may have been misnumbered;
- describe what they think I-12 might look like and where it might be located;
- given two intersecting interstates, predict the state in which they would intersect;
- identify the state in which two interstate highways, each numbered with perfect squares, would likely intersect; and
- identify other "mathematically interesting" intersections.



## 3. The Locker Problem

Present to students, who will work in groups, the following locker problem: In a certain high school there were 1,000 students and 1,000 lockers. Each year for homecoming the students line up in alphabetical order and perform the following strange ritual: The first student opens every locker. The second student goes to every other locker and closes it. The third student then goes to every third locker and changes it (i.e., opens closed lockers and closes open lockers). In a similar manner, the fourth, fifth, sixth, . . . students change every fourth, fifth, sixth, . . . locker. Obviously, following this pattern, the 1,000th and last student merely changes the condition of the 1,000th locker. After all 1,000 students have passed by the lockers, which lockers are open? Ask students to explain their reasoning and show the work they did to arrive at their answer. Also, ask students if they were able to identify a pattern to determine which lockers were left open.

## 4. The Cake Problem

Distribute pattern blocks to students and explain that a bakery makes and sells hexagonal cakes. The "basic" cake, represented by the yellow hexagonal pattern block, sells for $6 and feeds five people. The cost of larger cakes is calculated on the basis of this "basic" cake. For example, a birthday cake is made by surrounding the basic cake with a ring of six additional cakes. Ask students to construct the birthday cake and to determine what a birthday cake would cost and how many people it would feed.

Repeat this process for a graduation cake that is made by adding a ring to the birthday cake, and for a wedding cake that is made by adding a ring to the graduation cake. Determine the cost of these two new cakes and how many people each would feed.

Ask students to complete a table of their data and to identify the relationship between the number of rings and the cost of each cake, and between the number of rings and the number of people each cake can feed. Next, ask students to find formulas that describe these relationships and to use these formulas to find the cost and number of people who can be fed if one created a record-breaking cake with 100 rings! Finally, students should be asked to create graphs of these relationships and to explain why one graph is straight and the other one curved.

**As part of ongoing mathematics instruction in Grades 9-12, students should have instructional experiences like the following:**

## 1. The Olympic Long Jump

Students are given the following Olympic long jump winning distances.

| YEAR | MEN | WOMEN |
|---|---|---|
| 1948 | 25 feet 8 inches | 18 feet 8.25 inches |
| 1952 | 24 feet 10 inches | 20 feet 5.75 inches |
| 1956 | 25 feet 8.25 inches | 20 feet 9.75 inches |
| 1960 | 26 feet 7.75 inches | 20 feet 10.75 inches |
| 1964 | 26 feet 5.75 inches | 22 feet 2.25 inches |
| 1968 | 29 feet 2.5 inches | 22 feet 4.5 inches |
| 1972 | 27 feet 0.5 inches | 22 feet 3 inches |
| 1976 | 27 feet 4.5 inches | 22 feet 0.75 inches |
| 1980 | 28 feet 0.25 inches | 23 feet 2 inches |
| 1984 | 28 feet 0.25 inches | 22 feet 10 inches |
| 1988 | 28 feet 7.25 inches | 24 feet 3.5 inches |

Students are asked to analyze the data by looking for patterns, constructing a graph of the data, drawing some conclusions from the data and predicting when, if ever, they believe the women's winning long jump will be the same as the men's winning long jump.

Students also are asked to predict when the Olympic men's record will be greater than 30 feet.

## 2. Weighty Matter

Students are informed that the progress of three people on diets is recorded on the chart below.

| Weeks On Diet | Tom | Jaime | Rhonda |
|---|---|---|---|
| 0 | 210 | 158 | 113 |
| 2 | 202 | 154 | 108 |
| 4 | 196 | 150 | 105 |

Students are asked to examine the trends over the first four weeks of dieting and to predict each person's weight after six weeks on the diet and to explain their reasoning.

Students then are asked to use the data in the chart to construct a convincing argument that each one of the dieters is doing the best, so far, based on different ways of thinking about "best."

## 3. Polygon Investigation

Students are led in an investigation of the sum of the angles of polygons. First, use a protractor, then divide the polygons into triangles. Repeat the process using geometric computer tools to investigate polygons from triangles to octagons to n-gons by seeking and generalizing patterns.

## 4. Going Around In Circles

Students are presented with the following situation: The two congruent circles in Figure 1 are tangent to each other and to the x-axis at the points with x-coordinates 0 and 1.

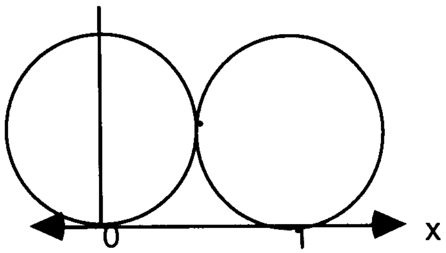



**Figure 1**

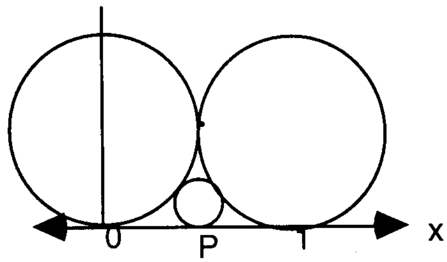



**Figure 2**

In Figure 2, a circle has been added to Figure 1. The small circle is tangent to each of the original circles and to the x-axis at point P. What is the x-coordinate of point P?

In Figure 3, circles have been added to Figure 2. The two new circles are tangent to the other circles as shown, and also tangent to the x-axis, at points Q and R. What are the x-coordinates of Q and R?

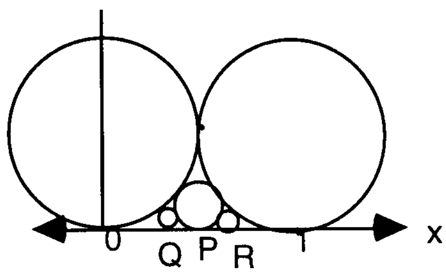



**Figure 3**

Students are asked to sketch their own version of Figure 4, adding four circles to Figure 3, with the new circles each tangent to two of the circles of Figure 3 and to the x-axis at points S, T, U, V. What are the coordinates of S, T, U, V? Guess a pattern. Show that your guess is valid.

## PROTOTYPE ASSESSMENTS AND SAMPLES OF STUDENT WORK

**As a result of an instructional program in mathematics like that described in this guide, by the end of Grade 4, all students should be expected to complete work like the sample below:**

### THE PATTERN MACHINE

A certain pattern machine turns wigglies into wobblies. Look at the table below and determine what the machine is doing to each wigglie to turn it into a wobblie. Whatever the pattern machine does to any one wigglie, it does to all wigglies.

| Wigglies | Wobblies |
|---|---|
| 3 | 8 |
| 0 | 2 |
| 6 | 14 |
| 5 | 12 |
| 7 | 16 |
| 1 | 4 |

1. What is the wigglie-wobblie rule for this pattern machine?

2. What would the pattern machine do to each of the following wigglies?

| Wigglies | Wobblies |
|---|---|
| 4 | ______ |
| 9 | ______ |
| 100 | ______ |
| 40 | ______ |
| 31 | ______ |
| 19 | ______ |

3. Create your own pattern machine rule and make up your own set of wigglies and wobblies. Share your wigglies and wobblies with a partner and see if he or she can discover your pattern machine rule.

**USE THIS PAPER FOR YOUR EXPLANATION OF: THE PATTERN MACHINE**

1. The Wiggly-Wobbly rule is that you: multiply by 2 then ~~and~~ add 2 to change a Wiggly into a Wobbly or subtract 2 then divide by 2 to change a Wobbly int a Wiggly.

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3.

| Wigglies | Wobblies |
|---|---|
| 54 | 20 |
| 108 | 50 |
| 135 | 65 |
| 36 | 10 |
| 90 | 40 |
| 171 | 85 |
| 207 | 105 |

To change a Wobbly into a Wiggly, you must first divide the Wobbly by 5. Then you add 2 to that number and then multiply by 9. To change a Wiggly into a Wobbly, first divide by 9, subtract 2, and multiply by five

**As a result of an instructional program in mathematics like that described in this guide, by the end of Grade 8, all students should be expected to complete work like the sample below:**

## THE RUMOR

A student at your school decides to start a rumor that the school has a serious safety flaw and is going to be shut down by the Building Department. On the first day the student tells two other students the rumor with the instructions that each of them is to spread the rumor to two more students the next day and that each of these new students is to spread the rumor to two more students on the third day, and so on. In other words, three students would have heard the rumor after the first day, four more hear it on the second day for a total of seven, and eight more hear it on the third day for a total of 15.

a. If the rumor continues to spread in this way, how many students will have heard the rumor on the 10th day?

b. If the rumor started on October 1 and if there are 8,000 students in the district, is it possible that all could have heard the rumor by October 14?

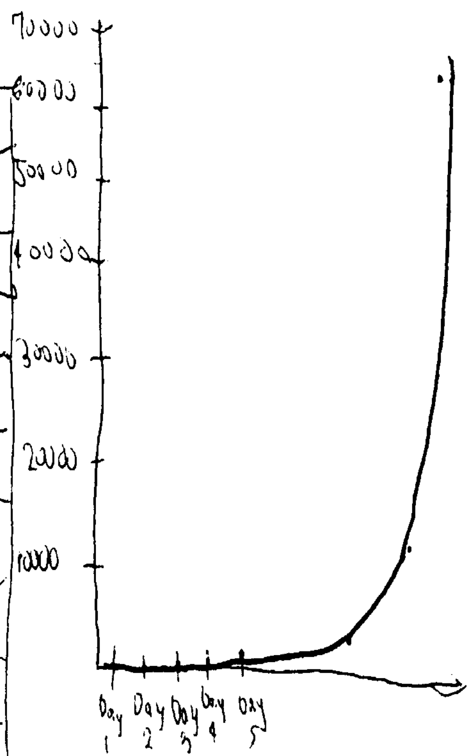
c. Construct a graph showing the number of students who could have heard the rumor by the end of each of the first 15 days of a month, assuming that the rumor was started on the first day of the month.

A. 2047

B. Yes all 8000 students would hear it.

C. 65536 students would of heard it.

| Days | | People |
|---|---|---|
| 1 | +2 | 3 |
| 2 | +4 | 7 |
| 3 | +8 | 15 |
| 4 | +16 | 31 |
| 5 | +32 | 63 |
| 6 | +64 | 127 |
| 7 | +128 | 255 |
| 8 | +256 | 511 |
| 9 | +512 | 1023 |
| 10 | +1024 | 2047 |
| 11 | +2048 | 4095 |
| 12 | +4096 | 8191 |
| 13 | +8192 | 16383 |
| 14 | +16384 | 32765 |
| 15 | +32768 | 65536 |



**As a result of an instructional program in mathematics like that described in this guide, by the end of Grade 12, all students should be expected to complete work like the sample below:**

### A VERY SQUARE PATTERN

An interesting thing happens if you add one to the product of any four consecutive positive integers.

Find the pattern, express the pattern in words and symbols, generalize the pattern, and show that it will work for all sets of consecutive positive integers.

A Very Square Pattern:

$1\cdot 2\cdot 3\cdot 4+1 = 24+1 = 25 = 5^2$ ) +6

$2\cdot 3\cdot 4\cdot 5+1 = 120+1 = 121 = 11^2$ ) +8

$3\cdot 4\cdot 5\cdot 6+1 = 360+1 = 361 = 19^2$ ) +10

$4\cdot 5\cdot 6\cdot 7+1 = 840+1 = 841 = 29^2$ ↓ +12

So: $5\cdot 6\cdot 7\cdot 8+1$ should equal $41^2$

$1680+1 = 1681$ It Does!!

So it appears that the ~~[illegible]~~ one more than the product of 4 consecutive positive integers equals a perfect square

$$(n)(n+1)(n+2)(n+3)+1 = k^2$$
$$(n^2+n)(n+2)(n+3)+1 = k^2$$
$$(n^3+3n^2+2n)(n+3)+1 = k^2$$
$$n^4+6n^3+11n^2+6n+1 = k^2$$
$$(n^2+3n+1)^2 = k^2$$

So if $n=5$, $k=41$ ✓

The pattern will work for all sets of 4 consecutive positive integers

## CONTENT STANDARD 9: Algebra and Functions

*Students will use algebraic skills and concepts, including functions, to describe real-world phenomena symbolically and graphically, and to model quantitative change.*

### K-12 PERFORMANCE STANDARDS

| Educational experiences in **Grades K-4** will assure that students: | Educational experiences in **Grades 5-8** will assure that students: | Educational experiences in **Grades 9-12** will assure that students: |
|---|---|---|
| • represent numerical situations using variables, expressions, equations and inequalities; and<br><br>• write and solve number sentences that describe real-life situations. | • use variables, expressions, equations and inequalities to describe and represent numerical situations;<br><br>• use concrete materials, tables, graphs, verbal rules and symbolic expressions to represent situations and patterns;<br><br>• analyze functional relationships to explain how a change in one quantity is associated with a change in another;<br><br>• construct and interpret data points on number lines and the coordinate plane; and<br><br>• solve simple linear equations using concrete, informal, graphical, tabular and formal methods. | • model and solve problems that involve varying quantities with variables, expressions, equations, inequalities, absolute values, vectors and matrices;<br><br>• model real-world phenomena using polynomial, rational, trigonometric, logarithmic and exponential functions, noting restricted domains;<br><br>• analyze the effect of parametric changes on the graphs of functions;<br><br>• translate among and use tabular, symbolic and graphical representations of equations, inequalities and functions;<br><br>• develop, explain, use and analyze procedures for operating on algebraic expressions and matrices; and<br><br>• solve equations and inequalities using graphing calculators and computers as well as appropriate paper-and-pencil techniques. |

## ILLUSTRATIVE TASKS AND EXPERIENCES

**As part of ongoing mathematics instruction in Grades K-4, students should have instructional experiences like the following:**

### 1. Square Numbers

Distribute color tiles, or one-inch-square ceramic tiles, or construct one-inch-square construction paper squares to explore the patterns of multiplication arrays. For example:

| | | | |
|---|---|---|---|
| • If 2 rows of 3 tiles each, or | □ □ □<br>□ □ □ | = | 2 X 3 = 6, what about 3 rows,<br>4 rows,... |
| • If 3 rows of 4 tiles each, or | □ □ □ □<br>□ □ □ □<br>□ □ □ □ | = | 3 X 4 = 12, |
| • Then 4 rows of 3 tiles, or | □ □ □<br>□ □ □<br>□ □ □<br>□ □ □ | = | 4 X 3 = 12. |

- What about 4 rows of 4 tiles?

Ask students to transfer their arrays to one-inch graph paper and have students cut them out. Discuss which shapes are rectangles and which are squares. Record findings by pasting the grid shapes into a booklet and writing the multiplication sentence that describes them.

### 2. Rosie The Robot

Draw on the chalkboard a robot with good-sized ears, and a keypad for "programming" rules for Rosie to use. Explain to the class that you programmed Rosie to follow a particular rule for any number you whisper in her ear. For example, add four to whatever number is whispered in, or double whatever number is whispered in. Pretend to program in a rule and tell Rosie a number by talking into her ear. Wait a few seconds, put your ear to Rosie's mouth and relay to the class the number Rosie "said." Continue this process with different numbers until the class begins to guess what rule Rosie is using. When you want to do a new program, simply clear out Rosie's keypad and put in a new program or rule.

To record the number whispered into Rosie's ear and the number she "says," draw a "T" grid on the board and fill it in as the lesson progresses.

### 3. Equations Galore

Students are asked to write equations for 8. For example 3+5=8. Encourage students to come up with as many different equations as they can. Then ask students to identify which equations are alike and why. Change the number and ask students to create different equations for the new number.

**As part of ongoing mathematics instruction in Grades 5-8, students should have instructional experiences like the following:**

## 1. Weight Loss

Students are asked to work in groups to solve the following problems and discuss alternative approaches to solving each one: Two different visitors to the Fitness Center arrived seriously overweight in the same week. One arrived weighing 230 pounds and has managed to lose 4.5 pounds each week. The other arrived weighing 195 pounds and has managed to lose 3 pounds each week.

- What was each dieter's weight after 10 weeks on the diet?
- After how many weeks did each of the dieters weigh 150 pounds?
- After how many weeks did the two dieters weigh the same?
- Explain why it is unlikely that real diets result in losing a constant amount of weight each week.

## 2. Accident Rescue

Students are presented with the following situation: There has just been a serious accident on a deserted stretch of the Interstate, 82 miles from the hospital in Western City and 209 miles from the hospital in Eastern City. A passing motorist's call to 911 is made at 1:14 p.m. The Western City hospital is served by an ambulance that can travel 70 miles per hour. The Eastern City hospital is served by a Life Star Helicopter that can travel 180 miles per hour. Ask students to work on the following problems:

- Assuming that both the ambulance and the helicopter require 10 minutes at the accident site, which vehicle will get the accident victim to which hospital's emergency room soonest?
- At what time would you expect the victim to arrive?
- Where on the Interstate would you be better served by the ambulance and where would you be better served by Life Star?

## 3. The Flaming Function

Each group of students is provided with a birthday candle. Ask students to do the following:

- Let x be the height of the candle in inches and y the elapsed time in seconds.
- Let the initial value of x be the height of the candle beginning where the candle flattens out.
- Mark the candle at quarter-inch intervals, starting at the top. Stand the candle on a heat-resistant surface. Light the candle.
- Let the candle burn to the first mark. Begin timing here. When the candle reaches the next mark record the time.
- Continue burning the candle and recording the time at each indicated mark. Repeat this until the candle burns to the last mark. Plot the ordered pairs (x, y) and analyze the data.

(Note: This activity is best done in a science lab.)

### 4. Amusement Park

Present the following situation to students: After exams, Juanita and her friends went to the amusement park. They found that each could buy an admission ticket for $5 and then pay 25 cents per ride. Their other option was to buy an admission ticket for $2 and then pay 75 cents for each ride.

- What factors should be considered in making a decision?
- Select one of the following methods:

  a. Use a graphing calculator to plot each option.
  b. Sketch a graph of each option, using the horizontal axis to represent the number of rides.
  c. Make a table with the number of rides as one column, and the cost of each option in successive columns.
  d. Write a system of equations and solve to find the number of rides that would cost the same under either option.
- Write a paragraph telling which would be the better deal and why.

## As part of ongoing mathematics instruction in Grades 9-12, students should have instructional experiences like the following:

### 1. The Algebra Walk

Set up a -5 to 5 number line on the floor with tape or chalk and place one student (a total of 11) on each of the integer points on the line facing "up" or above the number line. Explain that the number on which each is standing represents his or her first name. For example, the student standing on 3, has a first name of 3. Then explain that each student's last name is determined by a given rule. For example, your last name is three times your first name. Now have students move forward (positive) or backward (negative) to the point on the floor that represents their full name. For example, forward 9 paces to (3,9). Discuss and explore how and why this "multiply your first name by three" situation results in a tilted line, the line $y=3x$, a line with a slope of three, a line through the origin, etc. Now create other rules (for example, double your first name and add 4, take the absolute value of your first name and subtract 2, take the opposite of your first name and subtract 3) to develop a concrete sense of functions, slope, coefficients, intercepts, etc.)

### 2. Postage Stamps

Students are given the information that first class postage has increased from 3¢ in 1931 to 32¢ in 1994. Use large- and small-group instruction to discuss:

- various ways to describe this increase mathematically (up 26¢, up about 1/3¢ per year, up about 10 fold, an increase of 866 percent, up by some annual compounded growth rate);
- whether this increase is reasonable or unreasonable and why;
- what stamps would cost if the annual compounded growth rate were 5 percent;
- what the average actual growth rate has been and various approaches to determining this rate;
- how this growth rate compares to the rates of increase of other phenomena; and
- predictions about the cost of postage in 2001 and the reliability of such predictions.

## 3. The Coffee Urn

Bring to class a full 30-cup (or so) coffee maker. Gather data that compares the time (T) to fill a coffee cup with the height (H) of the coffee (or water) in the coffee maker. Analyze this data and arrive at a mathematical model that explains this relationship. Use the data to create a formula that expresses T in seconds as a function of H in inches. Discuss how well the selected model and the created formula match the original data and explain discrepancies.

## 4. Exploring Periodic Functions

Use a graphing calculator to explore the following trigonometric functions: $y=\sin x$; $y=\sin 2x$; $y=2 \sin x$; $y=2 \sin 2x$; $y=3 \sin x$; and $y=\sin 3x$.

- Write a paragraph discussing the graph of $y = A \sin Bx$ in comparison with the graph of $y=\sin x$. Do A and B have to be whole numbers?
- Sketch a graph of $y = 3 \sin 3x$ before using the calculator. Check your sketch by using the calculator.
- Explain whether or not the pattern seen here would hold for other functions.

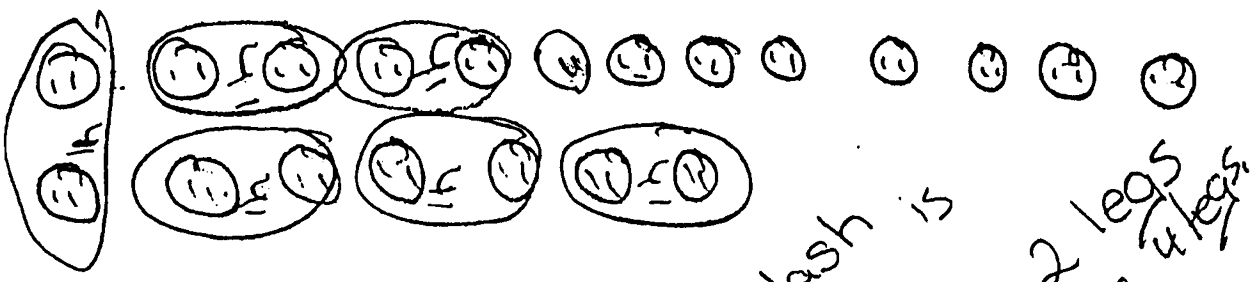
## PROTOTYPE ASSESSMENTS AND SAMPLES OF STUDENT WORK

**As a result of an instructional program in mathematics like that described in this guide, by the end of Grade 4, all students should be expected to complete work like the sample below:**

### HORSES AND CHICKENS

A farmer looks out into his barnyard and counts 14 animals – some horses and some chickens. He also counts a total of 40 legs among his animals. Can you figure out how many horses and how many chickens must have been in the barnyard?

You may work with a partner and use whatever materials you think will help you solve the problem.



6 horses
8 chickens

each slash is a leg. 1 chicken has 2 legs and a horse has 4 legs.

I knew that there were less than 10
horses (since a horse has 4 legs and 4x10 is 40 legs alre
and from that fact that there are more than 4 chickens
I built a table (pictured on the next page). I started
in the middle (7 horses and 7 chickens). That made 28+14=42 legs
which is too many. I knew that every time I took away a
horse and added a chicken, 2 legs were subtracted. Since
the difference between 42 and 40 is 2, I had to take
away a horse and add a chicken. That makes
24+16=40 legs which is what I'm looking for. So the
answer is 6 horses and 8 chickens

**As a result of an instructional program in mathematics like that described in this guide, by the end of Grade 8, all students should be expected to complete work like the sample below:**

## PICKING A PLAN

The phone company has two payment options as shown below.

**Plan #1:** \$1 per month plus 9 cents for each call
**Plan #2:** \$15 per month, 30 free calls and 17 cents per call after 30 calls

a) Suppose you must keep your monthly bill under \$20. How many calls can you make if you choose Plan #1? How many calls can you make if you choose Plan #2?

b) When does it make sense to choose Plan #1? When does it make sense to choose Plan #2?

c) Which of these plans would you choose for your family? Why?

$20 = 15 + .09x$

$5 = .09x$

$20 = 15 + .17x$

$5 = .17x$

Plan #1

It would make sense if you had many purposes to use your phone. In that case, it would be better economically to chose plan #1

Plan #2

I would chose plan #2 if I wanted to make less than or about 60 calls and I had a tight budget (under \$20)

For My Family

I would chose plan #1 because there are 5 people in my family and the phone line is used for pleasure and business.

**As a result of an instructional program in mathematics like that described in this guide, by the end of Grade 12, all students should be expected to complete work like the sample below:**

## TO COLLIDE OR NOT, THAT IS THE QUESTION

Two ships are sailing in the fog and are being monitored by tracking equipment. As they come onto the observer's rectangular screen, one ship, the Argonaut, is at a point 900 mm to the right of the bottom left corner of the screen along the lower edge. The other ship, the Bounty, is located at a point 100 mm above the lower left corner. One minute later, both ships' positions have changed. The Argonaut has moved to a position 3mm left and 2 mm above the previous location. Meanwhile, the Bounty has moved to a position 4 mm right and 1 mm above the previous location.

Assuming that both ships continue to move at a constant speed on their respective linear courses, explain, using graphs, tables and/or equations:

a. whether or not the two ships will collide; and
b. if so, when? If not, how close do the ships actually come to each other?

(Adapted from Pacesetter materials
with permission from the College Board, New York.)

2ND DRAFT

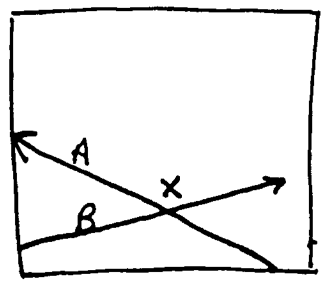
WILL THE SHIPS COLLIDE ?
(+ IF NOT, HOW CLOSE DO THEY GET ?)

a. GOOD NEWS! The ships will NOT COLLIDE!

The argonaut starts at (900, 0) + moves 3 left + 2 up
so the slope of its path is $-2/3$
Equation of Argonaut path: $y = -2/3x + 600$

The Bounty starts at (0, 100) + moves 4 right + 1 up
so the slope of its path is $1/4$
Equation of Bounty path: $y = 1/4x + 100$

Using my graphing calculator the paths look like:
Using Trace + zoom - The intersect
is (545.45, 236.36)




Since Bounty moves up 1 unit in each time period, it will
take $236 - 100 = 136$ Time units to get to point X
Since Argonaut moves left 3 units in each time period, it
will take $(900 - 545) \div 3 = 118$ time units to get to point Y

So the point of intersection is NOT a point of
collision since the 2 ships get there at
different times

b. The closest the ships will get is 28.28 nm after 128 time units.

This is because the coordinates of the 2 ships can be converted to parametric form

$$A: (900 - 3t, 2t)$$
$$B: (4t, 100 + t)$$

The distance formula tells you that the distance between A + B at any time t is:

$$D = \left[(900 - 7t)^2 + (t - 100)^2\right]^{1/2}$$

graphing this equation + finding the minimum reveals a minimum of

(128, 28.28)

That's when + how close the ships get.

So maybe a near miss - depending on the scale!

## CONTENT STANDARD 10: Discrete Mathematics

*Students will use the concepts and processes of discrete mathematics to analyze and model a variety of real-world situations that involve recurring relationships, sequences, networks, combinations and permutations.*

### K-12 PERFORMANCE STANDARDS

| Educational experiences in **Grades K-4** will assure that students: | Educational experiences in **Grades 5-8** will assure that students: | Educational experiences in **Grades 9-12** will assure that students: |
|---|---|---|
| • classify data according to attributes;<br>• solve simple counting problems;<br>• use diagrams and models of simple networks that represent everyday situations;<br>• identify and investigate sequences; and<br>• follow, devise and describe practical algorithmic procedures. | • use systemic listing and counting strategies, including simple combinations and permutations;<br>• use recursive processes, including iteration, to explore and solve problems; and<br>• devise, describe and test algorithms for solving optimization problems. | • represent problem situations using finite graphs, matrices, sequences and recurrence relations;<br>• develop, analyze, describe, invent and test algorithms;<br>• define and use permutations, combinations, mathematical induction and recursion to solve combinatorial and algorithmic problems; and<br>• understand and use appropriate strategies to solve optimization problems. |

## ILLUSTRATIVE TASKS AND EXPERIENCES

**As part of ongoing mathematics instruction in Grades K-4, students should have instructional experiences like the following:**

### 1. How Many Outfits?

Students are asked to imagine their grandmother or aunt sending them three new tops for their birthday – one yellow, one red and one green; and their grandfather or uncle sending them three new pairs of shorts – one yellow, one blue and one brown. Ask students to draw pictures or make a chart to show all the different combinations that can be made with these new clothes. How many days would it be before you would have to wear an outfit again?

**Extension:** Add different types of shoes or hats to these outfits.

### 2. Slide Or Jump

Students are taught to play the Slide Or Jump Game and, in groups, are asked to complete the following six-step investigation.

Using squared paper (1 cm squares will do) and some small red and blue counters, set up the game as shown below.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| B | B | B | B | B | | R | R | R | R | R |

This game is played with an equal number of red (R) and blue (B) counters on a line of squares. There is one more square than the total number of counters. The starting position for 5 red and 5 blue counters is shown above.

The object of the game is to reverse the position of the counters, i.e., to end up with

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| R | R | R | R | R | | B | B | B | B | B |

There are only two allowable moves. You may not move backward.

*Move 1*

You may *slide* a counter one square left or right onto an empty square, e.g., from the first position you could move to

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| B | B | B | B | | B | R | R | R | R | R |

*Move 2*

You may *jump* over a differenct colored counter onto an empty square, e.g. from the last position you could move to

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| B | B | B | B | R | B | | R | R | R | R |

Try playing the game. The target number of moves is 35. If you succeeded in 35 moves you did very well. If you didn't, don't worry, we'll get there eventually. Try playing with more or less than 5 counters of each color.

### 3. Sorting Classmates

Students are told that there are many ways people and things can be sorted. Tell students that each student is going to sort the class into groups and the class will try to guess the sorting rule that was used. (For example, students might be sorted by shoe type, or by color or type of clothing, or by some other characteristic that is visible.) Arrange the class in a central location and call on students who have a "sorting rule" to move their classmates into two or three clusters that everyone can see. Encourage guessing until the rule is discovered.

### 4. Careful Directions

Students are told that there are many things we do very naturally without thinking about how many sequential steps are involved to accomplish the task. For example, putting on a shirt, or drawing a particular figure, or making a peanut butter and jelly sandwich. Select a familiar task – like making a peanut butter and jelly sandwich – and ask students to think about the steps they would follow to make such a sandwich. Ask students to work in groups to devise specific instructions. Then, using peanut butter, jelly, bread and utensils, have students try out the instructions. Repeat this activity with other tasks.

## As part of ongoing mathematics instruction in Grades 5-8, students should have instructional experiences like the following:

### 1. Checkerboard Squares

Students are reminded that the standard checkerboard contains eight rows of eight squares, for a total of 64 little or 1x1 squares. Challenge students to find other sized squares on the checkerboard and then ask students to compute the total number of squares of all sizes on the board and to convince the class that they have counted all of the possible squares.

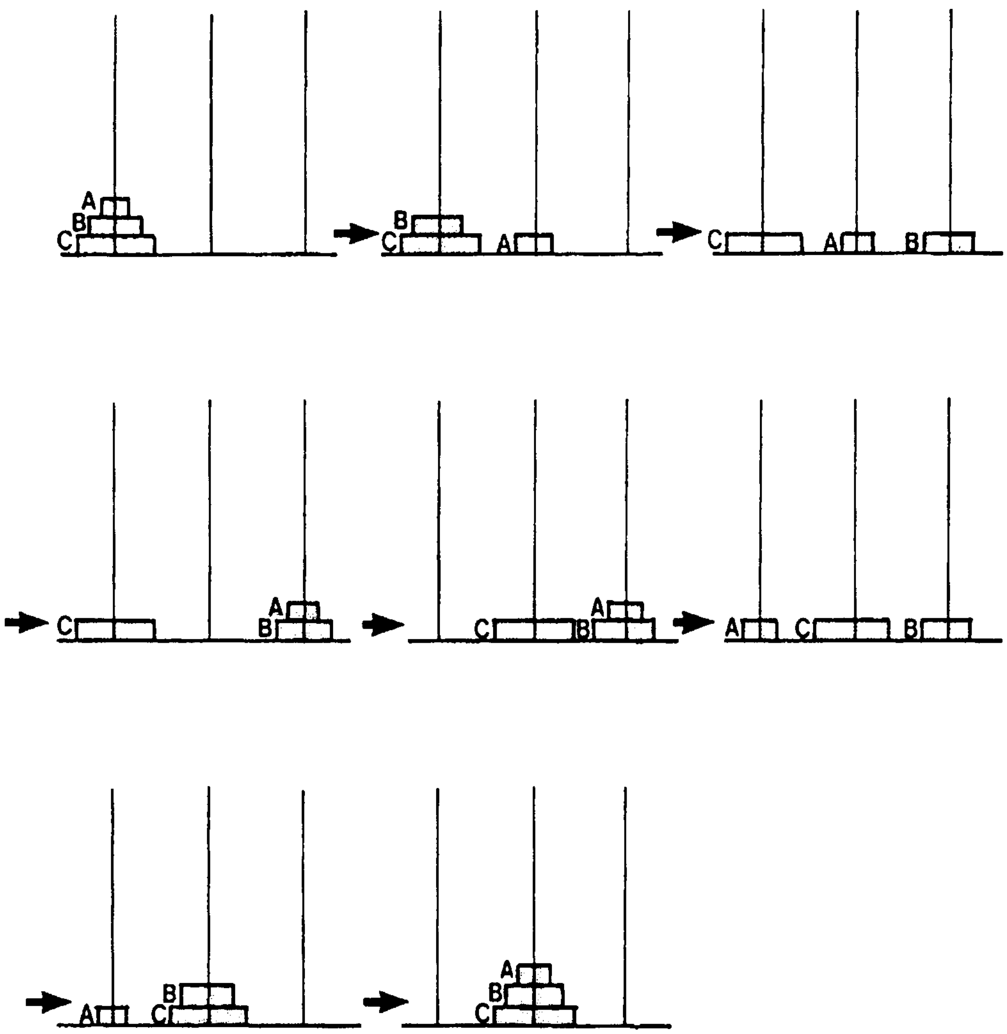
**Extension:** Try to find a pattern for the total number of squares in a checkerboard with 1, 2, 3,... small squares on each side.

### 2. Towers Of Hanoi

Students are provided with the rough details of the Towers of Hanoi problems. Ask students to build a table and generalize about the minimum number of moves for any number of discs. Then explain the legend of the 64-disc tower that tells us that, at one second per move, the world would end when the last of the 64-disc tower is placed on a new pole.

# Towers Of Hanoi

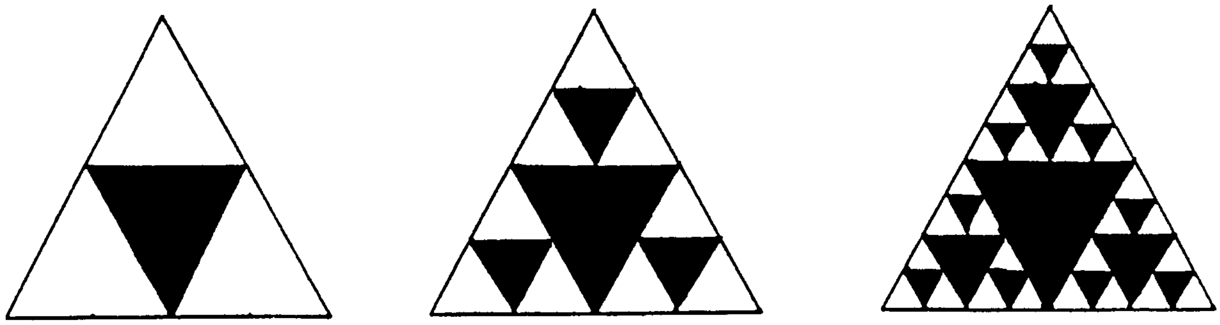
The Towers of Hanoi consist of three poles. On one pole are stacked a number of discs each of smaller diameter than the one below. The discs have to be moved one at a time to end up stacked on another pole. At no time may a disc be placed on top of a smaller disc. For 3 discs the moves would be



**In this investigation you are asked to consider other numbers of discs and to describe any patterns which you find in the numbers of moves required in each case.**

### 3. The Sierpinski Triangle

Students are shown the first, second and third iteration of the Sierpinski Triangle.



Ask students to explore the patterns of the successive triangles by:

- drawing a picture of the 4th Sierpinski triangle;
- calculating the ratio of shaded area to total area in the first four triangles; and
- calculating the perimeters of the shaded triangles in the first four triangles.

### 4. Computer Searches

Explain to students that there are two common ways of finding one particular object in a set. They can check each item individually to see if it is the one they are searching for, OR they can successively cut the set of objects in half until they find the one they are searching for. Demonstrate these two search strategies by selecting a number from 1 to 40. See how long it takes to find the selected number with random guesses vs. a "more or less than 20" strategy. Explain how binomial – successive cutting in half – searches are used to locate names from directory assistance and from other databases. Ask students what is the maximum number of guesses needed to find one name from a list of 100; 1000; 10,000; 100,000 and 1 million.

## As part of ongoing mathematics instruction in Grades 9-12, students should have instructional experiences like the following:

### 1. Truck Refuse

Explain to students that a refuse-hauling company brings sealed containers of recycled aluminum and glass to the recycling center.

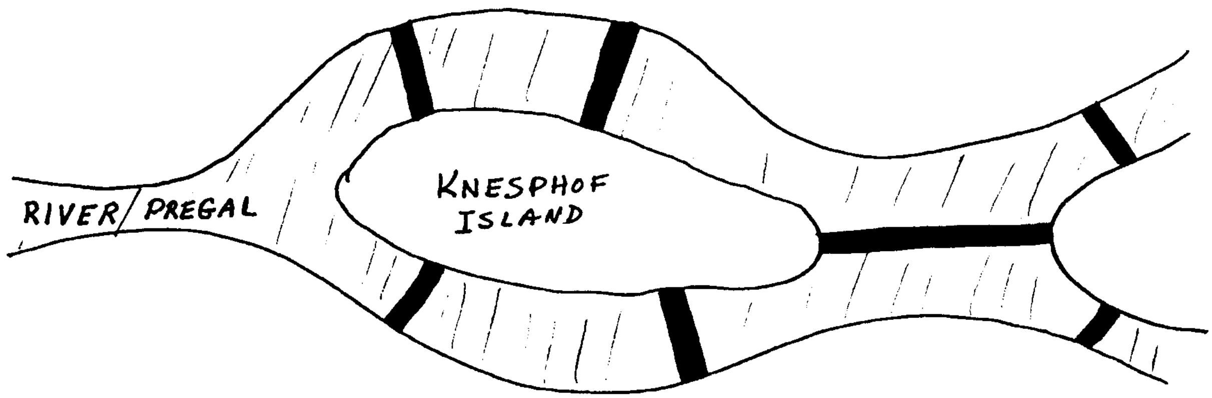
- Each aluminum container weighs 800 pounds and occupies 60 cubic feet.
- Each glass container weights 900 pounds and occupies 75 cubic feet.

The company charges $120 for hauling each aluminum container and $95 for each glass container. If each of the company's trucks has a weight limit of 22,000 pounds and a maximum of 1740 cubic feet of cargo space, determine the company's maximum income for a single load of recycled glass and aluminum.

## 2. The Seven Bridges Of Konigsberg

Relate to students the famed tale of the City of Konigsberg, where the River Pregal and its two islands split the city in two and where seven bridges connect the islands and the mainland. A resident of the Knesphof Island wondered whether she could take a walk that crossed *every* bridge only *once* and happily return home. Ask students to explore whether such a path exists. Suppose a resident of the mainland also wishes to cross each bridge once and return to his starting point. Is this possible. Suppose one of the bridges is closed. Is such a walk over the remaining six bridges possible?

Use the Konigsberg situation to create a graph – lines and vertices – and explore the relationships of possible paths and the presence of odd and even vertices. (See sketch below.)




## 3. Plus Growth Vs. Times Growth

Explain to students that a great uncle's will leaves them \$100 and the offer of either an additional \$10 each week OR an additional 10 percent of their holdings each week for a year. Use tables, graphs and the recursive formulas: Pn+1 = Pn + 10 and Pn+1 = Pn x 0.10 to determine which offer is better. Change the conditions and explore situations for which "plus some number" and "times some percent" are the same.

## 4. Candy Sale

Explain to students that members of the school's jazz band, symphonic band and orchestra are selling candy for a fund-raiser. Each group has sold gum balls, sour balls, chocolate mints and peppermint patties. Make up sales numbers between 100 and 500 for the sales to date of each candy type for each musical group. Ask students to use matrices to record the sales to date for each group and each type of candy. Then ask students to explore the use of matrices to answer questions like the following:

- If each group has a goal of 1,000 of each type of candy, how many of each type of candy must each group still sell?
- If the four candies sell for 14¢, 18¢, 9¢ and 15¢ each, respectively, how much have the three groups earned so far?
- If a similar fund-raiser was conducted last year, compare the sales of this year to those of last year.

## PROTOTYPE ASSESSMENTS AND SAMPLES OF STUDENT WORK

**As a result of an instructional program in mathematics like that described in this guide, by the end of Grade 4, all students should be expected to complete work like the sample below:**

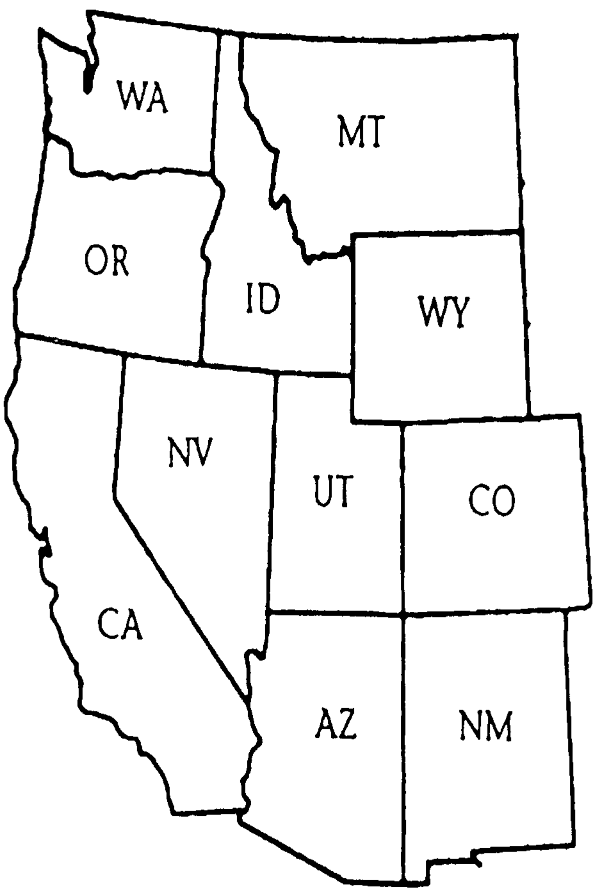
### MAP CHALLENGES

A cartographer is an individual who makes maps. Using colors on maps is very expensive, so mapmakers try to use as few colors as possible.

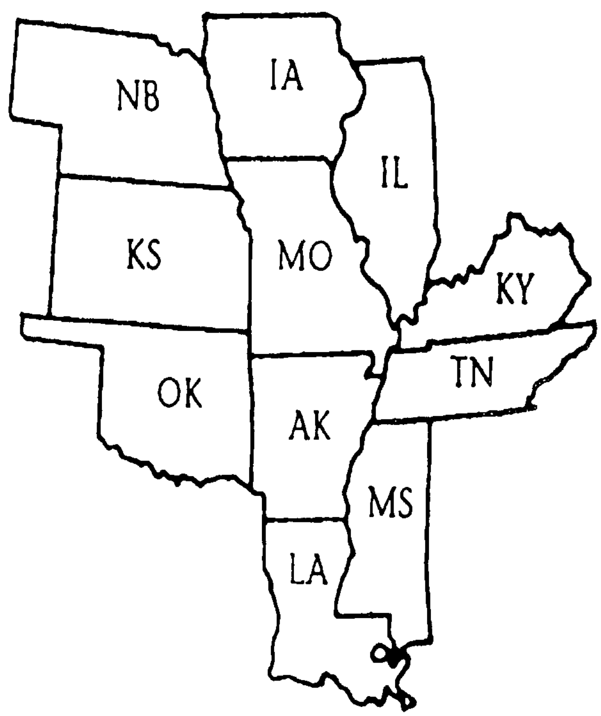
Map-Coloring Rules

- Use as few colors as possible.
- Areas that share a border may not be the same color.
- Areas that meet at a point (or corner) may be the same color.

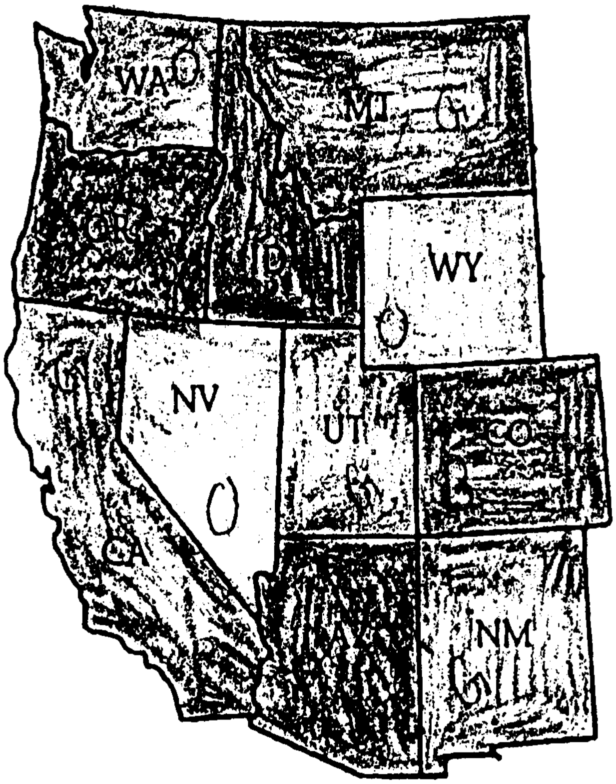
Look at the two partial USA map designs provided. Try to help the cartographer choose the fewest colors possible. Write the number of colors you need for each design next to the design. Then color the design.




a.

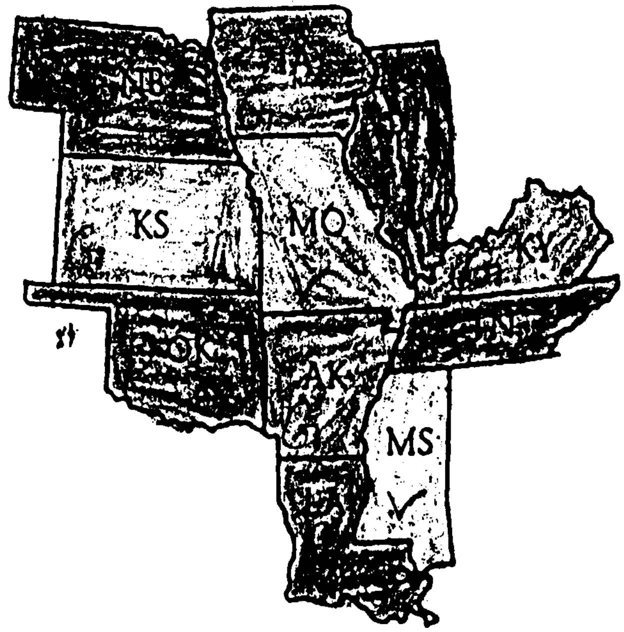



b.




a.

G=green=4 green
B=Blue=3 blue
O=Orange=3 orange
P=purple=1 purple




b.

R=red=5 red
G=green=4 green
V=violet=2 violet

**As a result of an instructional program in mathematics like that described in this guide, by the end of Grade 8, all students should be expected to complete work like the sample below:**

## PHONE TREES

In case of an emergency radiation leak at a nearby nuclear power plant, a community is planning to set up a phone tree. The original plan specifies that the plant superintendent calls three key community contact persons who, in turn, each call three persons, who then each call three additional persons, and the tree continues until all 10,000 households have been reached.

1. Draw a diagram and/or table showing how many rounds of calls must be made before everyone is notified.
2. If the superintendent makes the first call at 10:21 and if each call takes 1 minute, at what time will the last household be notified?
3. One member of the community is concerned that it might take too long to notify everyone and proposed a five-call tree. That is, the superintendent calls five persons and each person in turn calls five others. If, once again, each call takes 1 minute, how long would it take for the five-call-tree system to reach all 10,000 households?
4. Propose a phone tree system that would notify all 50,000 households in a community in less than 15 minutes.

# P.O.W.
## Problem of the Week

In our most recent problem of the week, we are asked a question concerning phone trees. A community set up a phone tree to notify the citizens of a possible nuclear meltdown. The supervisor of the plant would call tree citizens, who in turn would each contact three more citizens. This cumulative chain would continue until all 10,000 households were reached. We are asked to draw a diagram or table displaying how many calls it would take to notify everyone. Also, we are asked to find at what time everyone would be notified if the calls begin at 10:21 and take one minute each. Another question we must answer is how long it would take to notify everyone using a five-call tree if the calls take one minute each. I am then asked to create a phone tree system that could warn a community of 50,000 in 15 minutes or less.

To prepare to solve these problems, I gathered a pencil, paper, calculator, and my POW statement sheet. When I first read the problem, I felt it would be very difficult to answer. I have never done a problem similar to this one. For all the problems, I am assuming that you can go over the specified amount of people you need to contact (you don't have to contact exactly 10,000 or 50,000 people, you can go over this number). For the first problem, I will make a chart with columns labeled total calls, people per round and time. I will increase the people per round by powers of 3 and adjust the total calls and amount of time accordingly. For the second problem, I will take this same chart and find the total time it took to notify the citizens. I will take this time and appropriately add it to the time of 10:21. For the third problem, I will make a chart similar to the one I used to solve the first problem. I will make columns labeled people per round, total calls, and time. The only difference is that I will increase the people per round by powers of 5 instead of powers of 3. For the last problem, I will use guess and check to see if there is a system that could inform 50,000 households in 15 minutes. For the first problem, I estimate that there will be ten rounds of calling before all the citizens are notified. For the second problem, I guess that everyone will be notified by 10:50. In the third problem, I feel it will take 30 minutes to notify every one of the 10,000 households. For the last problem, I don't think it is possible to notify all of the 50,000 of the households in 15 minutes.

Everything I tried worked, but some plans needed a little modification. I got help from Mrs. Hahn. She helped me with the chart so that I could get started on the right path. The chart I constructed for the first problem is on the attached sheet of paper. I found it took nine rounds to notify all the citizens of a nuclear meltdown. The last round wasn't a full round, but it was still a round (actually less than a third of a round). For the second problem, you can also check the chart on the attached page. The chart would indicate that it takes 27 minutes to notify all the people, but actually it takes less. Since my totals go over the 10,000 people specified, you can notify all the people with two less calls. With only one call in the last round instead of three, you can still notify all the 10,000 people. By taking away these last two calls, you save two minutes. The whole community will be notified 25 minutes later at 10:43. My work for the five-call tree is on the attached page and it will take 27 minutes to warn all of the people. Even though the chart indicates 30 minutes, you can save three calls and still notify everyone. For the last problem, I was unable to find a system that could notify all the people in 15 minutes. I don't think it's possible.

For the first problem, I found it took 9 rounds, even though the last round wasn't full. For the second problem, the town will be notified at 10:43. For the third problem, it will take 28 minutes to notify everyone. For the fourth problem, I found that there isn't a way to notify 50,000 people in 15 minutes. All of answers make sense because I have checked them over and work is correct. There are no other correct answers to this problem. All of my answers were close to my original guesses. I learned that if I have trouble on a POW or other problem my teacher will help. This was a difficult problem and it was very time consuming.

Problem 1 & 2

# Attached Sheet

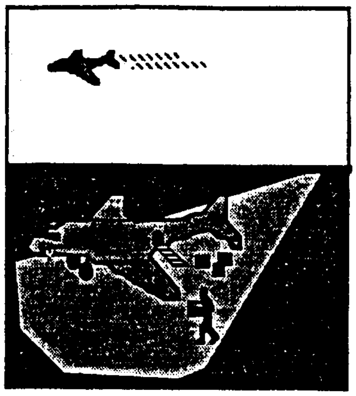
| Time (min) | People per Round | Total calls |
|---|---|---|
| 0 | 1 | 1 |
| 3 | 3 | 4 |
| 6 | 9 | 13 |
| 9 | 27 | 40 |
| 12 | 81 | 121 |
| 15 | 243 | 364 |
| 18 | 729 | 1093 |
| 21 | 2187 | 3280 |
| 24 | 6561 | 9841 |
| 27 | 19,683 | 29524 |

Problem 3

| Time (min) | People per round | Total calls |
|---|---|---|
| 0 | 1 | 6 |
| 5 | 5 | 31 |
| 10 | 25 | 156 |
| 15 | 125 | 781 |
| 20 | 625 | 3 |
| 25 | 3125 | 3906 |
| 30 | 15,625 | 19531 |

**As a result of an instructional program in mathematics like that described in this guide, by the end of Grade 12, all students should be expected to complete work like the sample below:**

## HEAVY CARGO



Linda Sue is a stunt pilot, but she has found she can't make a living doing stunts. So she has bought a transport plane from her friend Philip. He agrees to help her set up her business.

He has two customers he no longer has time to serve and suggests that she work for them, delivering their merchandise.

*Charley's Chicken Feed* packages its product in containers that weigh 40 pounds and are 2 cubic feet in volume. Philip has been charging them $2.20 a container.

*Careful Calculators* packages its materials in cartons that weigh 50 pounds and are 3 cubic feet in volume. Philip has been charging them $3.00 a carton.

The plane can hold a maximum volume of 2000 cubic feet of cargo and it can carry a maximum weight of 37,000 pounds.

*Charley's Chicken Feed* and *Careful Calculators* have both told Linda that she can have as much business as she can handle, and the same flight route serves both customer's needs. Of course, Linda Sue wants to maximize the money she gets per flight, so she can spend more time on stunt flying.

Here are your tasks:

1. Use variables and algebra to describe the constraints on what Linda Sue can carry.
2. Sketch the feasible region for Linda Sue's situation.
3. Find the combination of chicken feed and calculators that Linda Sue should carry in order to maximize her income.

# Heavy Cargo

In this problem a person by the name of Linda Sue is a stunt pilot but realizes she can't make a living being just a stunt pilot. She decides to transport packages in her spare time. There are two types of packages she can deliver: Charley's Chicken Feed packages and Careful Calculator packages. She makes $2.20 per Charley's Chicken Feed container and $3 per Careful Calculator carton. Her plane can hold 2,000 cubic feet of cargo and it can carry a maximum of 37,000 pounds, these are called the constraints of the problem. Each Charley's Chicken Feed container weighs 40 pounds and takes up 2 cubic feet. Each Careful Calculator package weighs 50 pounds and takes up 3 cubic feet. Linda Sue has to decide how much of each type of cargo she wants to carry in order to maximize the money she gets for each flight.

In order to solve this problem I had to graph this situation out. Before you graph a problem like this, you have to turn the constraints into equations in the y= format. In this problem there were two constraints: the amount of weight the plane can carry and the amount of cubic feet the plane can hold. I assigned the x value to Charlie's Chicken Feed and the y value to Careful Calculators. The two equations I got out of those constraints were 40x + 50y =< 37,000 and 2x + 3y =< 2000. I changed these both into y= format and graphed each of the equations on graphing paper. I found the feasible region on the graph, the area where all combinations of x and y fit the constraints. Within the feasible region I found the point that had the maximum profit, which was (550, 300). So, for Linda Sue to get the most profit out of her work she would need to transport 550 containers of Charley's Chicken Feed and 300 cartons of Careful Calculators. This would give Linda Sue $2110 per flight.

Heavy Cargo

x = Charlie's Chicken Feed y = Careful Calculators

Weight Constraint

$40x + 50y \le 37000$

$50y \le 37000 - 40x$

$y \le (37000 - 40x)/50$

Volume Constraint

$2x + 3y \le 2000$

$3y \le 2000 - 2x$

$y \le (2000 - 2x)/3$

Profit

$2x + 3y$