

## 6. The Middle School Mathematics Project:

Probability - Activities III and IV, pp 29-42 and 79-90

Spatial Visualization - Activity II, pp 36-48

Mouse and Elephant: Measuring Growth - Activities III and IV,  
pp 29-58

Similarity and Equivalent Fractions - Activities III and IV, pp 33-61

Factors and Multiples - Activities I and II, pp 1-33

7. The Geometry Menu is a menu approach to a hands-on study of geometry with a problem solving focus. See The Math Solution.

## 8. Algebra Activities - What's My Rule, The Function Machine, What Comes Next, etc.

## 9. Various games and activities that promote development of number sense, logical thinking, etc.

Digit Place	NDQ (No Dumb Questions)
Guess My Number Riddles	People Sorting
Too High/Too Low	Bicycle Problems
Who Has?	Creature Cards
Base 10 Riddles	Guess My Rules
\$ Riddles w/Too High Too Low	Scattered Numbers
Rounding Riddles	Arrow Math
20 Questions	Match Game (Factors)
Buzz	Path Game
Reject	\$1.00 Words
Mystery Stories	Around the World

10. Get It Together is an excellent resource of activities for cooperative group problem solving.

## 11. OTHER RESOURCES

Moving On With Geoboards

Tangram Activities

Pentominoes

TOPS Deck - Level B

Calculator TOPS Deck

Addison-Wesley Mathematics Textbook - Grade 5

rev. 2/95

**NCTM STANDARD 10: MEASUREMENT**

*In grades K-4, the mathematics curriculum should include measurement so that students can-*

- . understand the attributes of length, capacity, weight, area, volume, time, temperature, and angle;*
- . develop the process of measuring and concepts related to units of measurement;*
- . make and use estimates of measurement;*
- . make and use measurements in problem and everyday situations*

**BY THE END OF GRADE 2, THE STUDENT WILL:**Outcome

1. Experience non-standard units of measure.
2. Have experiences with measurement vocabulary using the attributes of length, volume, weight, area, time, and temperature when describing their physical environment and their own bodies.
3. Estimate and measure length in meters, centimeters, yard, feet, inches
4. Estimate and measure weight and capacity using balance scales and large and small containers.
5. Use measurement in comparing, counting, estimating, and making one-to-one correspondence.
6. Tell time to the nearest hour, half-hour, and quarter hour.
7. Sequence days of the week and months of the year.
8. Participate in a variety of writing and speaking experiences using measurement vocabulary.
9. Solve a variety of problems involving measurement.
10. Begin to select an appropriate non-standard or standard unit of measure for length.

Sample Instructional Activities

- Measure items in room using clothespins, milk containers, paper clips, string, etc.
- Guessing Jar: Students take turns filling a jar with some object from home (m&m's, crackers, paper clips, etc.) and other students guess how many handfuls fit in jar, then how many objects. Students filling jar write in journal about their choice, how many, etc...
- Estimate how long it takes for some observable event to occur (ex: how long it takes a cup to sink to bottom of pail of water). Use non-standard measure: How many stars I can draw before...; How many times I can stand up, sit down before... How many circles can I draw in 1 minute?

Recommended Materials

unifix cubes, balance and weight scales, toothpicks, clothespins, paper clips, rulers, meter and yard sticks, containers for volume measure, clocks, thermometers

**NCTM STANDARD 10: MEASUREMENT**

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- . make and use estimates of measurement;*
- . make and use measurements in problem and everyday situations.*

**BY THE END OF GRADE 4, THE STUDENT WILL:**Outcome

1. Select, estimate, and use appropriate non-standard units of measure.
2. Make comparisons and estimate using:  
volume in liters, ounces, pints  
quarts, gallons  
weight in grams, kilograms, pounds  
ounces, tons  
length in meters, centimeters, kilometers, yards, feet, inches, miles  
angles greater than, less than or equal to a right angle.
3. Measure volume, length, weight to the nearest  $\frac{1}{2}$  unit.
4. Select, estimate, and use appropriate units of measure within a system of linear measure.
5. Find perimeter of a regular or an irregular polygon.
6. Find the area of a rectangle and express it in square units.
7. Read and write and measure time in the following ways:
  - a. to the nearest minute (digital and analog); to the month, day, and year
  - b. describe elapsed time - clock and calendar;
  - c. know equivalents - minutes/hour, hour/day, days/week, days/month, days/year

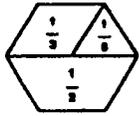
Sample Instructional Activities

- Reinforce use of measurement & selecting appropriate measurement (standard and non-standard) in other subjects: Art students design and lay out bulletin board Science - students choose appropriate measures for an experiment; Social studies - students make maps, with a scale, of some thing or place well known, and then compare to actual map & scale.
- Encourage measurement of objects and distances outside the classroom (playground, hallways, bus routes, etc...)
- Build into problem solving hidden information problems which require knowledge of measures and equivalents in order to solve.
- Use television scheduling and after school activities to determine elapsed time. Have students plan their own weekly schedules.
- Write a how-to letter to someone describing a measurement problem including a description of something like a bedroom. Use precise terms and accurate measurements and units.

8. Integrate manipulative usage and appropriate measurement vocabulary effectively communicate measurement information to others.
9. Solve problems involving measurement.
10. Begin to read and estimate and have a sense of temperature (Celsius & Fahrenheit) in relation to hot & cold.

### Recommended Materials

double sided tape measures, rulers, meter sticks, yard sticks, balance scales and weights, containers for volume measure, thermometers, clocks, rolling meter measurer, yarn or string, Base 10 blocks, geoboards and elastics, maps.

**Cluster—Rational Numbers****GRADE 6****Strand—Estimation, Fractions, Decimals, Percents**

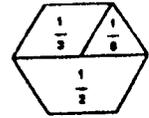
Rational numbers describe the relationships of parts to a whole. Students must develop a variety of ways to think about, model, describe, and write about the relationships. Fractions, decimals, and percents are different notation systems for the same idea—ratio relationships. The concept of equivalence helps students understand rational numbers, select the most efficient notation to represent a rational number, and make good estimates of reasonable answers to problems.

Through cutting and comparing real objects, children develop an understanding of whole and parts which leads to the conceptual exploration of fractions that are part of the whole. By counting out equal subsets and comparing them, students explore ratio and the concept of parts of a set (i.e.,  $\frac{1}{3}$  of 12 is 4 and  $\frac{2}{3}$  of 12 is 8). Patterns connect ratio concepts to multiplication and division.

Students engage in talking about the parts in relation to the whole and writing about them in different ways. They should not focus on the algorithms for manipulating the notation, but instead on the way fractions, decimals, and percents describe real objects or pictures of objects. Students need continuous experience using a variety of manipulative materials in order to develop the base of understanding that will support them as they develop ways to compute with fractions, decimals, and percent. Without these models, students mindlessly manipulate symbols, merge the steps in operations, or forget them. With the consistent use of models that support visualization, students are able to construct their own strategies, make more reasonable estimation, and verify their answers. Fractions, decimals, and percents have been combined into a single strand because they are natural and easier to teach in connection with one another.

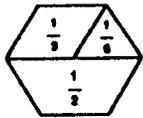
The essential topics for instruction in fractions include naming fractions, comparing fractions, equivalence of fractions, and operations with fractions. It seems that fractions with numerators of 1 do not pose much of a problem for students. Yet when fractions such as  $\frac{2}{5}$ ,  $\frac{3}{4}$ , and so on enter the picture, so does confusion. Also, children have difficulty deciding which of two fractions,  $\frac{2}{3}$  and  $\frac{4}{5}$ , for example, or even  $\frac{1}{8}$  and  $\frac{1}{16}$ , represents more, and why fractions such as  $\frac{3}{6}$  and  $\frac{4}{8}$  describe the same part of a unit. These types of difficulties indicate the need for further concept development with concrete materials and real-life situations.

When teaching students who lack basic understanding of fractions, it's not uncommon for teachers to feel the pressures of time and the demands of the curriculum. So, it's often a temptation to speed up instruction and to teach the rules for operating with fractions. Some teachers resort to telling students, for example, that multiplying the numerator and denominator by the same number gives an equivalent fraction, or that in order to divide you invert the fraction on the right and multiply across the tops and across the bottoms.



### Working Towards CMT Objectives on the Grade 8 Mastery Test

2. Relate fractions, decimals, and percents to their pictorial representations.
3. Rename fractions and mixed numbers as equivalent decimals and vice versa.
4. Rename fractions and decimals as equivalent percents and vice versa.
5. Identify points on number lines, scales, and grids including fractions, decimals, and integers.
6. Estimate the magnitude of mixed numbers and decimals.
7. Add and subtract 2-, 3- and 4-digit whole numbers, money amounts, and decimals.
8. Multiply and divide 2- and 3-digit whole numbers, money amounts and decimals by 1-digit whole numbers and decimals.
9. Multiply and divide whole numbers and decimals by 10, 100, and 1000.
10. Add and subtract fractions and mixed numbers with reasonable and appropriate denominators.
11. Multiply whole numbers and fractions by fractions and mixed numbers.
12. Find percents of whole numbers.
14. Identify an appropriate procedure for making estimates involving fraction and mixed number computation.
15. Identify an appropriate procedure for making estimates involving decimal computation.
16. Identify an appropriate procedure for making estimates involving percents.
17. Solve problems involving order and magnitude of fractions.
18. Solve problems involving order and magnitude of whole numbers and decimals.
19. Solve problems involving rounding whole numbers and decimals.
24. Solve or estimate a reasonable answer to problems involving fractions, decimals, and mixed numbers.
25. Solve or estimate a reasonable answer to problems involving ratios, proportions, and percents.



**Cluster—Rational Numbers**

**GRADE 6**

**Strand—Estimation, Fractions, Decimals, Percents**

Giving students rules to help them develop facility with procedures will not help them understand the concepts. The risk is that when students forget a rule, they have no way to reason through a process. Try this assessment with your students: Give students a fraction problem they're "supposed" to understand,  $1/2 + 1/3$ , for example. Ask them to show you what the problem means with any concrete material, or a drawing, or by relating it to some real-life situation. Listen to students' responses, and let their responses guide you when making instructional choices.

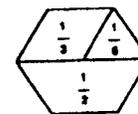
It is important to provide a variety of ways for students to learn about fractions—with concrete materials, from a geometric perspective, with a numerical focus, and related to real-life situations. Let the students know that different people learn in different ways, and that they should pay attention to the kinds of activities that help them develop understanding. Encourage them to try activities with which they are less comfortable. Do not expect immediate results from any one activity. Students need time to absorb new ideas and integrate them with the understanding they already have.

**TEACHING**

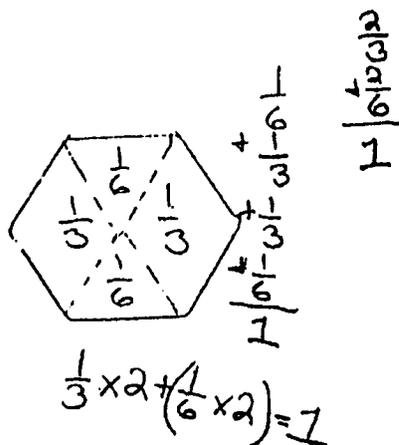
Objectives	Activities
<p>6.40 Compare, order, and round fractions and decimals. (See also 6.44). <i>CMT Grade 8 Objectives 5, 6, 17, 18, 19.</i></p>	<p>Have students construct, use, describe, draw and write about models and diagrams which show the value of fractions and provide a measurable model for comparison. For example, students may model fractions by folding paper strips to represent unit fractions with denominators from 1 through 10 and compare the lengths. Students may use the number line as a diagram to locate and order fraction and decimal values.</p>

**Cluster—Rational Numbers**

**GRADE 6**



**Strand—Estimation, Fractions, Decimals, Percents**



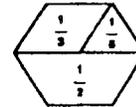
A student's model of addition and multiplication with pattern blocks.

**STRATEGIES**

Materials and Resources	Assessment
<p>6.40 Use:</p> <ul style="list-style-type: none"> <li>pattern blocks</li> <li>paper strips</li> <li>paper squares</li> <li>place value blocks</li> <li>rods and squares</li> <li>ten-strips and 10 x 10 grids</li> <li>tangrams</li> <li>geoboards</li> <li>number lines</li> <li>rulers</li> </ul> <p>See:</p> <ul style="list-style-type: none"> <li><i>Family Math</i>, "Fractions Kit", 120-123.</li> <li><i>Math in Stride, Book 6</i>, "Ordering Fractions and Decimals," Teacher Edition (TE) 112 and Student</li> </ul>	<p>Ask students to demonstrate, describe, draw and write about models that will convince someone that:</p> <ul style="list-style-type: none"> <li>a) <math>1/2 = 6/12</math></li> <li>b) <math>2/3 &gt; 2/4</math></li> <li>c) <math>0.5 &gt; 0.2</math></li> <li>d) <math>3/5</math> is closer to <math>1/2</math> than <math>3/4</math> is.</li> </ul> <p>See also <i>Math in Stride Performance Assessment, Student Book 6</i>, pages 26-27, 62-64, 75, and 77.</p>

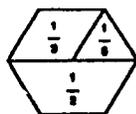
**Cluster—Rational Numbers**

**GRADE 6**



**Strand—Estimation, Fractions, Decimals, Percents**

Materials and Resources	Assessment
<p>Workbook (SW) 164-165.  <i>Math in Stride, Book 6</i>, "Ordering Decimals" TE 117 and SW 170.</p>	
<p>6.41 Use:                      pattern blocks                      place value blocks                      decimal squares                      fraction bars                      fraction circles</p> <p>See:  <i>Math in Stride, Book 6</i>, "Equivalent Expressions for Fractions, Decimals, and Percents," TE 64-66 and SW 94-103.  <i>Math in Stride, Book 6</i>, "Simplifying Multiplication of Fractions with Prime Factors," TE 109-111 and SW 157-163.  <i>Math in Stride, Book 6</i>, "Multiplication of Decimals as Multiplication of Fractions," TE 131-133 and SW 182-196.</p>	<p>Have students choose, draw, label and write about models that will convince someone that:</p> <p>a) <math>2/4 + 2/8 = 3/4</math>                      b) <math>1/2 + 1/3 \neq 2/5</math>                      c) <math>1/2 + 1/10 = .50 + .10</math>                      d) Ask students to show at least three different ways to find <math>4 \times 3-1/2</math>.                      e) Ask students to draw a model that shows the answer to the problem: <math>1/4 \times 3-1/2</math>.</p>
<p>6.42 Use:                      pattern blocks                      rulers                      paper folding                      place value blocks, strips and grids                      Explorer calculators</p> <p>See:  <i>Family Math</i>, "Judy's Fractions," page 125.  <i>Math in Stride, Book 6</i>,</p>	<p>See the assessment tasks suggested in the Sample Lesson.</p> <p>See also <i>Math in Stride Performance Assessment, Student Book 6</i>, pages 7, 30, 50, and 75-79.</p>

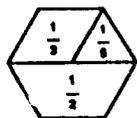


**Cluster—Rational Numbers**

**GRADE 6**

**Strand—Estimation, Fractions, Decimals, Percents**

Objectives	Activities
<p><b>6.41</b> Use multiple strategies to identify equivalent fractions and decimals, and to add, subtract, and multiply with them.                      (See also 6.44).  <i>CMT Grade 8 Objectives 2, 3, 4, 7, 10, 11, 14, 15.</i></p>	<p>Extend the use of models and diagrams and the comparing and ordering of fractions and decimals to the identification of equivalent values, and estimation and computation. For example, use pattern blocks to prove that <math>1/3 = 2/6</math> and that <math>1/2 + 1/3 = 5/6</math>.</p> <p>Use a 10 x 10 grid to show that <math>20/100 + 5/100 = 25/100</math> (or <math>1/4</math>) and <math>.20 + .05 = .25</math> (or <math>1/4</math>).</p>
<p><b>6.42</b> Build and draw models to demonstrate the concepts of multiplication and division with fractions and decimals.                      (See also 6.44).  <i>CMT Grade 8 Objectives 2, 6, 8, 9, 11, 14, 15, 24.</i></p>	<p>See the <i>Clinton Sample Lesson—Connecting the Strands—An Investigation of Division with Fractions</i>. This lesson helps students develop an understanding of multiplication and division with fractions and helps them connect the use of patterns, estimation, and spatial models of length and area. The lesson moves from conceptual understanding through models, diagrams, and real-world applications to working with fraction</p>



**Cluster—Rational Numbers**

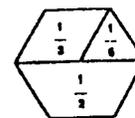
**GRADE 6**

**Strand—Estimation, Fractions, Decimals, Percents**

Objectives	Activities
	<p>notation and patterns with the symbols, to using patterns with the Explorer calculator to find and use the reciprocal.</p>
<p><b>6.43</b> Write and solve equations using mixed numbers, like and unlike fractions, and decimals that match word problems involving addition, subtraction, and multiplication. (See also 6.44). <i>CMT Grade 8 Objectives 7, 8, 9, 10, 11, 14, 15, 17, 18, 19, 24.</i></p>	<p>Begin by having students write a story problem that could be solved using either of the following equations:</p> $1/2 + 3/4 = x$ $.50 + .75 = x$ <p>Repeat with other equations. Then turn the process around and have students write and solve equations to solve practical problems, such as finding the amount in <math>3\text{-}1/4</math> boxes of candy bars if one dozen is in a box.</p>
<p><b>6.44</b> Use, describe, and write about ways to estimate and identify reasonable answers to problems involving rational numbers. (See also 6.40-6.43). <i>CMT Grade 8 Objectives 6, 14, 15, 17, 18, 19, 24.</i></p>	<p>Build an estimation step into the classroom discourse as students solve problems involving comparing, ordering, rounding, adding, subtracting, multiplying and solving story and practical word problems with fractions and decimals. Encourage students to predict reasonable answers and describe a variety of ways to</p>

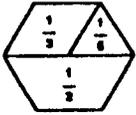
**Cluster—Rational Numbers**

**GRADE 6**



**Strand—Estimation, Fractions, Decimals, Percents**

Materials and Resources	Assessment
<p>“Multiplication With a Model” and “Division With a Model,” TE 59-62 and SW 88-91.</p> <p><i>Passport, Professional Handbook</i>, Section H 92-H95.</p> <p><i>Passport, Student Text Lab 8.4</i>, pages 370-371, and practice pages 372-375, and 377-385.</p> <p><i>Math Land</i>, “Interpreting and Solving Problems Involving Multiplication and Division of Fractions,” pages 126-133.</p>	
<p>6.43 Use: Student selected models and diagrams, as needed.</p> <p>See: <i>NCTM Standards</i>, “Math as Problem Solving,” pages 75-77, “Math as Communication,” pages 78-80, and “Number and Number Relationships,” pages 87-90.</p>	<p>See <i>Math in Stride Performance Assessment, Student Book 6</i>, pages 56-57 and 66-68.</p> <p>Also, have students create their own story problems and have other students try to solve them.</p>
<p>6.44 See: Objectives 6.40-6.43.</p>	<p>Ask students to contrast two different ways to estimate an answer, and then use each way to do a different, but similar problem. For example, <math>0.12 \times 3.52</math> could be about <math>\approx 1/10 \times 3-1/2 \rightarrow 1/10 \times 3-5/10</math></p> <p>or</p>

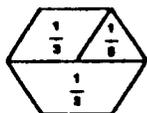


**Cluster—Rational Numbers**

**GRADE 6**

**Strand—Estimation, Fractions, Decimals, Percents**

Objectives	Activities																														
	<p>estimate the answer to a problem.</p>																														
<p><b>6.45</b> Build and draw models to show equivalences of fractions, decimals, and percents.  <i>CMT Grade 8 Objectives 2, 3, 4, 5, 6, 17, 18, 19, 24, 25.</i></p>	<p>Use the 10 x 10 grid drawn on centimeter graph paper and shade in some of the centimeter squares. For example, if 15 squares are shaded, you have 15 out of 100, or 15/100, or 0.15, or 15%.</p> <p>Shade in 25 squares and have students write as many symbols as they can to describe the diagram. For example, they might propose 25/100, 0.25, 1/4, and 25%.</p> <p>Experiment with other amounts of shaded centimeter squares.</p>																														
<p><b>6.46</b> Explore using equivalence to find simple percents of a number.  <i>CMT Grade 8 Objectives 2, 16, 25.</i></p>	<p>Set up a table and look for patterns to help fill it in. For example:</p> <table border="1" style="margin: 10px auto; border-collapse: collapse; text-align: center;"> <thead> <tr> <th>n/100</th> <th>a/b</th> <th>Percent</th> <th>of 100 Objects</th> <th>of 50 Objects</th> </tr> </thead> <tbody> <tr> <td>1/100</td> <td>1/100</td> <td>1%</td> <td>1</td> <td>1/2</td> </tr> <tr> <td>2/100</td> <td>1/50</td> <td>2%</td> <td>2</td> <td>1</td> </tr> <tr> <td>3/100</td> <td>3/100</td> <td>3%</td> <td>3</td> <td>1 1/2</td> </tr> <tr> <td>4/100</td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td>etc.</td> <td></td> <td></td> </tr> </tbody> </table> <p>Repeat with other amounts, such as 200, 60, 30, or 20 objects.</p> <p>Have students describe the patterns.</p>	n/100	a/b	Percent	of 100 Objects	of 50 Objects	1/100	1/100	1%	1	1/2	2/100	1/50	2%	2	1	3/100	3/100	3%	3	1 1/2	4/100							etc.		
n/100	a/b	Percent	of 100 Objects	of 50 Objects																											
1/100	1/100	1%	1	1/2																											
2/100	1/50	2%	2	1																											
3/100	3/100	3%	3	1 1/2																											
4/100																															
		etc.																													



**Cluster—Rational Numbers**

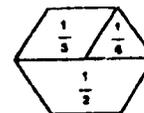
**GRADE 6**

**Strand—Estimation, Fractions, Decimals, Percents**

Objectives	Activities
	<p>Set up a data-gathering project and connect graphing the data onto a circle graph with estimating simple percents. For example, try the activities from the NCTM Addenda Series cited in the next column.</p>

**Cluster—Rational Numbers**

**GRADE 6**

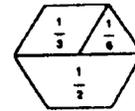


**Strand—Estimation, Fractions, Decimals, Percents**

Materials and Resources	Assessment
	<p><math>\approx 0.10 \times 3.5 \rightarrow 0.35</math></p> <p>Ask students to choose and defend what they think is the most efficient (easiest) method to estimate.</p>
<p>6.45 Use: place value blocks 10 x 10 grids</p> <p>See: <i>CT Mastery Test Handbook—Second Generation—Math</i>, the section on equivalence. <i>Math in Stride, Book 6</i>, “Fraction and Percent Equivalence,” TE 114 and SW 167.</p>	<p>See performance tasks in <i>Math in Stride Performance Assessment, Student Book 6</i>, pages 31, 56, 58, and 75-80.</p>
<p>6.46 Use: grids and tables calculators computer spreadsheet/graphing/packages with circle graphs.</p> <p>See: <i>NCTM Addenda Series—K-6 Sixth Grade Book</i>, “Making Sense of Data,” pages 16-18. <i>NCTM Addenda Series—Grades 5-8—Understanding Rational Numbers and Proportions</i>, “Investigation 3: Garbage and</p>	<p>Have students gather some data and display it using the circle graph—approximating and sketching by hand. Then have students use a computer software package to construct and print a circle graph from the same data and write a description of the interpretation of each graph.</p> <p>See also performance tasks in <i>Math in Stride Performance Assessment, Student Book 6</i>, pages 56-57, 65, 80, and 107.</p>

**Cluster—Rational Numbers**

**GRADE 6**



**Strand—Estimation, Fractions, Decimals, Percents**

Materials and Resources	Assessment
<p>Recycling: The Rational (Number Approach,” pages 32-48.  <i>Math in Stride Book 6</i>, “Fractional Parts of a Percent,” TE 134 and SW 197 and “Choosing Strategies to Solve Percent Problems,” TE 135 and SW 198-199 and “Solving Percent Problems,” TE 136 and SW 200-201.</p>	

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# TRIGONOMETRY

## A #143

In Trigonometry, students apply a unified, holistic, problem-solving technique to measure indirectly and to project and design solution systems to represent real world periodic relationships.

### The students will:

1. **Perform basic operations with angles including measurement in degrees and radians, rotations, and co-terminal angles.**

Concrete areas to which these concepts can be applied:

- . functions and relations,
- . distance formula,
- . arc length,
- . co-terminal solutions,
- . using scientific calculators and graphing calculator,
- . changing from degree measure to radian measure,
- . changing from radian measure to degree measure,
- . angular and linear velocity,
- . area of a sector.

2. **Define and evaluate the trigonometric functions of an angle.**

Concrete areas to which these concepts can be applied:

- . predicting values of trigonometric functions,
- . determining values using tables,
- . special cases (30-60-90) and (45-45-90),
- . using scientific and graphing calculator,
- . define functions using the unit circle.

3. **Solve right triangles and related real world problems using trigonometric functions.**

Concrete areas to which these concepts can be applied:

- . indirect measure using the trigonometric of a right triangle,
- . angles of elevation and depression,
- . word problem applications in navigation, surveying, and construction using line of sight and bearings,
- . scientific calculators.

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**4. Graph the trigonometric functions and their inverses on a Cartesian coordinate system.**

Concrete areas to which these concepts can be applied:

- . amplitude, periods, phase shift, vertical shift, and symmetry,
- . establishing identities and relationships,
- . rotations about axis and transformations,
- . graphing calculator and computers,
- . predicting results,
- . composite functions.

**5. Use fundamental trigonometric identities to write equivalent trigonometric expressions and prove other identities.**

Concrete areas to which these concepts can be applied:

- . substitution,
- . sum and difference of two angle measures identifies,
- . double angle identities,
- . half angle identities,
- . product and sum identities,
- . graphing calculator and computers to verify and analyze results,
- . solving trigonometric equations.

**6. Use the laws of sines, cosines, and tangents to solve triangles and find their area.**

Concrete areas to which these concepts can be applied:

- . oblique triangles,
- . ambiguous case of triangles,
- . Heron's formula,
- . word problem applications in navigation, surveying, and construction using line of sight and bearings,
- . scientific calculators.

**Define and convert polar coordinates and vectors.**

Concrete areas to which these concepts can be applied:

- . graphing calculators,
- . operations of complex numbers in polar form,
- . converting rectangular coordinates to polar coordinates,
- . converting polar coordinates to rectangular coordinates,
- . developing and predicting graphs,
- . parametric functions,
- . applications of vectors.

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8. (optional) **Define the use of harmonic motion sequence and series in application and solutions of patterned projections.**

Concrete areas to which these concepts can be applied:

- . composite functions,
- . graphing calculators and computers,
- . predicting results,
- . establishing identities and relationships.

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**TWO SAMPLE CRITERION-REFERENCED ASSESSMENTS**

1. **November Mathematics Assessment Test – Grade 3**, Pages 207-218.  
From the Simsbury, Conn. Public Schools  
Used with permission from: Rosanne Daniele, Frances Solomon and Nancy Robbins.
2. **Basic Algebra I Final Exam**, Pages 219-229.  
From Nonnewaug High School, Woodbury, Conn.  
Used with permission from James Paniati.