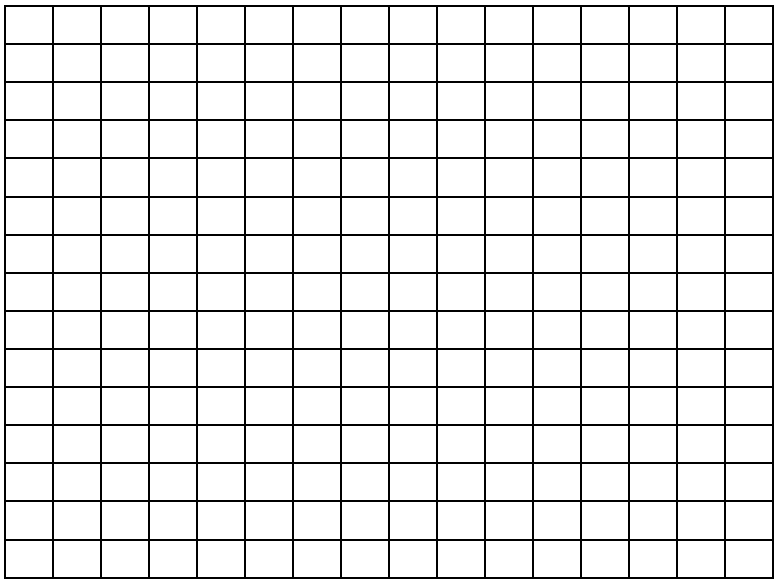
GARAGES AND PHONES

A local parking garage charges $6.00 for anytime during the first hour of parking. Each additional hour, or any part thereof, costs $2.40. Plot the cost of parking a car as a function of the time that the car is parked in the garage.

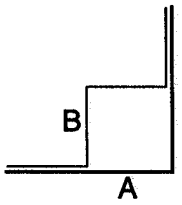
A small telephone company charges a $6.00 flat monthly fee independent of the amount of calls customers make. The telephone company charges for usage at the rate of $2.40 per hour. Fractional parts of an hour are billed accordingly (for example, one half hour costs $1.20, fifteen minutes costs $0.60, one minute costs $0.04, and so on). Plot monthly telephone costs as a function of the time that the phone is used.

Discuss the similarities and differences of your graphs.

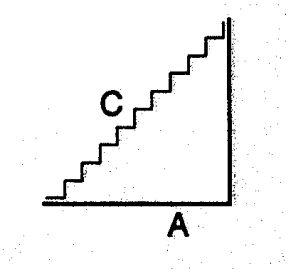


Extension:

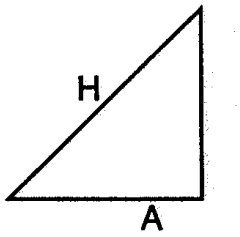
Here is a puzzle that is related to this task:



Curve A is just as long as curve B.



So is curve C.

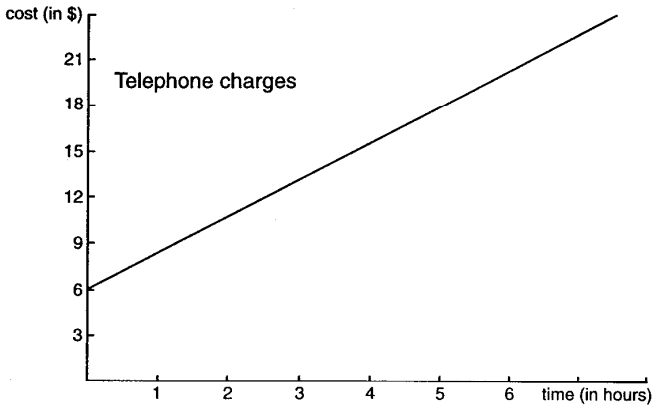


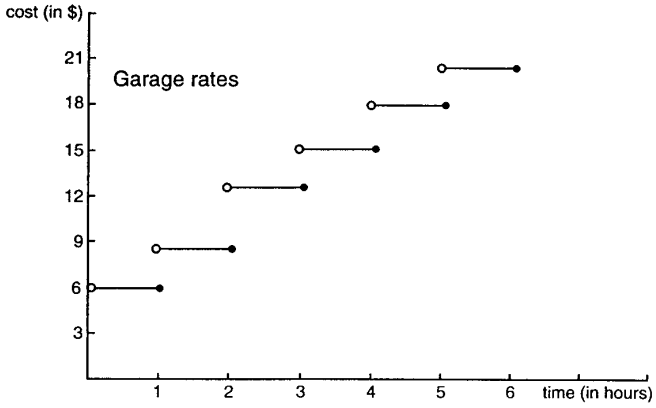
If you continue making the steps smaller, the staircase gets arbitrarily close to the hypotenuse H, but H is smaller that A! Explain why this is true.

A Sample Solution

Task 5

The essential point in this problem is for students to display an understanding of the difference between a linear function and a piece-wise constant function. Below are the two graphs for this problem.





The cost in dollars of telephone usage is

$$6 + 2.4T,$$

where T is in hours.

The cost in dollars of parking in the garage is

$$6 + 2.4 \text{ int}(T),$$

where T is in hours and int(T) denotes the integer part of the value of T.

Task 5

A line drawn from the point (0 hours, $6) through the point (5 hours, $18) will be identical on both the garage rates and the telephone charges graphs.

The point on the left edge of each horizontal segment in the garage rates graph corresponds to a point on the telephone charges graph.

It is possible to define a slope for the telephone charges graph but not for the garage rates graph.

The domain of each function is continuous.

The range for the garage function consists of isolated points separated by $2.40. The range of the phone function would have been continuous if it could charge fractions of a cent, but in reality the range is isolated points, starting at 6 and separated by $0.01.

For any given amount of time, it is never more expensive to park in this garage than it is to talk on the telephone.