Name Date

Keep Taking the Tablets

This problem gives you the chance to

- *investigate a situation systematically*
- *make tables and sketch graphs*
- *make and test generalizations*
- *give written and symbolic explanations*

Doctors have to plan the amount of medication they give very carefully. If they give too little, it won't work. If they give too much, they may harm the patient. If a patient takes a tablet every hour, this is what happens:

Start here.
There is no drug in the body.

↓

Take a tablet.
The amount of drug in the body increases by 32 units. So,
ADD 32
How much is in the body now?
Write this number down.

↓

One hour later,
The body gets rid of half the drug. So,
DIVIDE BY 2
How much is in the body now?
Write this number down.

↑ (back to "Take a tablet.")

Time in hours	Amount of drug in the body just **before** the tablet is taken	Amount of drug in the body just **after** the tablet is taken
1	0	32
2	16	48
3	24	56
4	28	60

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Name Date

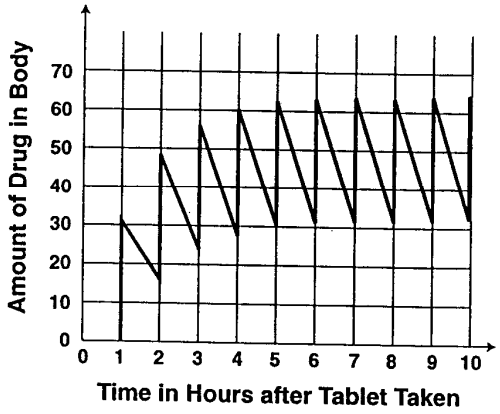
1. a. Copy the table shown on the previous page and extend it up to 10 hours.
   b. Describe how the amount in the body changes with time. Use words and a quick sketch graph. (You do not need graph paper for this.)
   c. What would happen if the patient continued taking tablets every hour for, say, 1000 hours? (Just make a reasonable prediction.) Explain why you think this.
2. Investigate what happens for other drugs and doses. (This means changing the 32 and the 2 to other numbers.) Try to find patterns, make predictions, and find general rules. Try to explain *why* your rules work using words and algebra.

Task 9 A Sample Solution

1a.

Time in hours	Amount of drug in the body just **before** the tablet is taken	Amount of drug in the body just **after** the tablet is taken
1	0	32
2	16	48
3	24	56
4	28	60
5	30	62
6	31	63
7	31.5	63.5
8	31.75	63.75
9	31.875	63.875
10	31.9375	63.9375

1b. The quantity of drug in the body just before the tablet is taken increases toward a limiting value of 32 units and 64 units just afterwards. Students will find that, due to rounding on their calculators, these limits may appear to be reached. Theoretically, though, they are approached asymptotically.



1c. After 1000 hours or so, the figures will be extremely close to, but not quite equal to 32 units and 64 units.

Task 9

2. In general, suppose a dose of size d is administered every hour. (In this case $d = 32$). Then the amount of drug in the body, D, just before the second dose will be rd (for some $r < 1$). In this example, $r = 0.5$. Just after the second dose, the amount will be $rd + d$ or $d(1 + r)$. Before the third dose, then, D will be $r \times d(1 + r)$, and after the third dose, $r \times d(1 + r) + d$, or $d(1 + r + r^2)$.

   Eventually, the quantity of drug eliminated from the body each hour becomes almost equal to the dose size and the drug level will reach its maximum level, D_{max}, where $D_{max} = r D_{max} + d$.

   This implies $D_{max} - rD_{max} = d$ and therefore $D_{max} = \frac{d}{1-r}$.

   This is the simplest derivation of the general formula, but it does assume that the limits will be reached.

   Alternatively, after successive doses the maximum amount of drug in the blood will be:

   d,
   $d(1 + r)$,
   $d(1 + r + r^2)$
   $d(1 + r + r^2 + r^3)$
   ...
   $\frac{d(1 - r^{n-1})}{1 - r}$

   and as $n \to \infty$ this approaches $\frac{d}{1-r}$ since $r < 1$.

   We anticipate that most students will approach the task by systematically varying d and r, making tables and spotting patterns. These would then lead to generalizations that may be tested and verified empirically. Only the most able would be able to use algebra.