Exploring Two Kinds of Probability

Introduction

Objective → Students will be able to describe the difference between mathematical probability and probability based on data.

Context → Students have worked informally with probability. They understand how to form simple fractions and what fractions mean. Students will continue to determine probabilities in situations with increasing numbers of outcomes.

NCTM Standards Focus

Students encounter probability situations in their everyday lives and basic probability concepts can be explained by connecting to these experiences. In this standards-based lesson, students engage in a hands-on probability experiment designed to generate a significant amount of data. This will enable students to compare mathematical and experimental probabilities and to recognize the relationship between the two.

Reasoning and Proof Students will analyze the situation of flipping a coin and explain how they determined mathematical probability. They will compare what they expect to happen with experimental results and generalize about the two kinds of probability.

Connections The connection between experimental and mathematical probability is a key focus of the lesson. Students will rely on prior knowledge about fractions to express chance as a measure. By connecting to real-life situations, students will reinforce their understanding of the fundamental concepts of probability explored in this lesson.

Communication Students will discuss how chance is involved in different situations. They will communicate their understandings of probability concepts both orally and in writing.

TEACHING PLAN

Materials → Student pages A.119-120; pennies

INTRODUCE THE LESSON by discussing familiar situations in which people talk about the chance of something happening. Encourage students to give examples of situations that involve change. These might include a weather reporter talking about the chance of rain, a sports team being given a certain chance of winning a game, familiar board or computer games, or

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television shows where a contestant spins a wheel.

Explain that probability is a measure of chance and that in this lesson they will be looking at different situations to determine the probability of something happening.

Ask students if they know what is meant by a "fair" coin. Make sure they understand that a fair coin is one with heads on one side and tails on the other and that, when flipped, is equally likely to land on either side. *If you flip a fair penny, how many different ways can it land? Explain.* (2; it can either land with heads up or with tails up.) *Do both have the same chance of happening?* (Yes.) Make clear that because getting heads on a coin flip is one of two possible things that can happen, we say the probability is 1 out of 2 (or 🗁). *What is the probability of getting tails?* (Also 🗁)

Explain that when we find a probability by thinking about what should happen, the probability is called ***mathematical probability***. Sometimes we find a probability by using information from actual experience. The process of gathering this information is called an ***experiment*** and consequently, this second kind of probability is called ***experimental probability***.

HAVE STUDENTS WORK in pairs to explore both kinds of probability by performing their own experiments. Distribute a copy of student page A.119 and a penny to each pair of students. Direct their attention to the table for Round 1 on the worksheet and explain that they are to flip a penny 10 times and record the *outcome* each time by making a mark in either the heads or tails column. When they have completed all 10 flips, they are to enter the total for each column at the bottom.

Before beginning the experiment, have students predict the results they will get. *What do you think the total for heads will be for your 10 flips? Tails? Why?* (Students may say that for half of the outcomes heads should come up and for the other half tails should come up, so the results should be 5

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heads and 5 tails.) Instruct students to write their predictions for both heads and tails under each column and then proceed with the experiment.

After students have completed Round 1, discuss their findings. Make sure all groups summarize their results. *How many heads did you get? How many tails? How did your results compare to your prediction? If the results were very different from your prediction, why do you think this is so?* (Many students will not have a response, but some may suggest that more tries are needed.)

f.y.i.

Experimental Probability

$$P\text{(event)} = \frac{\text{number of favorable outcomes}}{\text{total number of trials}}$$

Remind students of how the mathematical probability of 🗁 for getting heads or tails was determined. Explain that the experimental probability is found by comparing the number of favorable outcomes with the total number of flips of the coin. The experimental probability for getting heads would be the number of times heads actually came up over the total number of flips (in this case, 10). Have students find the experimental probabilities for Round 1 and compare their results with the mathematical probabilities. *Does a mathematical probability of 🗁 mean that something will always happen 🗁 of the time?* (No.) *Can mathematical and experimental probabilities have different values?* (Yes.) *What does mathematical probability tell you?* (What should happen.) *What does experimental probability tell you?* (What actually happened.)

Ask students to offer predictions for outcomes for Round 2. Encourage students whose predictions and outcomes differed considerably in Round 1 to explain their thinking for predicting what might happen in Round 2.

What Students Might Say

- A student who had a 7 to 3 split might predict a 3 to 7 result for Round 2, reasoning that maybe the outcomes will even out.
- A student with a 9 to 1 split might predict another 9 to 1 split, thinking that perhaps his or her coin is not fair.
- A student with a 5 to 5 split might think he or she completed Round 1 correctly and will repeat the performance.

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Have students complete Round 2 and discuss their results by answering questions similar to those for Round 1. Point out that while different groups may have gotten different individual results from Round 1, a similar overall variation between predicted and actual results occurred. Then let students move on to Round 3.

AFTER STUDENTS HAVE COMPLETED Round 3, have them explore how having a larger number of coin flips and outcomes affects the comparison between the two kinds of probability. Instruct students to find the total number of heads and the total number of tails for all three rounds. *If the probability was exactly* 🗁*, how many heads should you have?* (15) *How many tails?* (15) *Are the total results closer to the mathematical probability that those for each separate round?* You may also wish to combine results for the whole class to produce numbers that are even closer to 🗁.

Summarize the important ideas of the exploration. Emphasize that as students just saw, what should happen (mathematically) usually does not occur. Explain that as the number of times an experiment is repeated gets very large, for example one thousand or even one million times, the predicted results and the actual results should get closer. Use the class totals to justify this statement. You might want to encourage students to give intuitive explanations of why this is so. (They may suggest that lots of tries allows the results to "even out" or "balance out.")
If time allows, close the lesson by posing a situation for students to consider. *You flip a penny. It lands heads. The next time you flip the coin, is the probability of getting heads less than, equal to, or greater than* 🗁? *Explain your reasoning*.

Students may think the chance of getting heads has been "used up," so the probability for getting heads is less. Or, they may think the first flip of heads shows you are more likely to get heads again. Explain that the penny doesn't "remember" the last flip, and the probability of 🗁 does not change.

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Student Pages

Student page A.119 provides recording charts for the penny-flipping experiment. Page A.120 presents students with mathematical and experimental probability situations to be analyzed and explained.

Assessment

While students worked on their penny-flipping experiments, it was possible to assess their understanding of outcomes and probability and their ability to make predictions. You were able to observe their ability to apply this learning as they analyzed their experimental data and compared the predicted and actual results.

NCTM Standards Summary

In this lesson, students flipped pennies to explore the concepts of mathematical and experimental probability. Reasoning was employed as students used their understanding of these concepts to analyze and identify the type of probability being used in different situations. The activities in the lesson helped students to make the connection between mathematical and experimental probability and express this relationship in their own words. Students relied on prior knowledge about fractions to write and compare probabilities and linked the concept of chance to everyday situations. Throughout the lesson, students clarified their thinking about probability as they used written and oral communication.

Answers

Page A.119

Answers will vary.

Page A.120

1. 4 outcomes; red, blue, green, yellow
2. Each outcome has the same chances because the sections for each color are equal.
3. ▤; mathematical probability
4. ▤
5. 0; There are no purple spaces on the spinner.
6. 100; ▤ of 400 = 100
7. 20 times
8. 10 times
9. 10/20 = 1/2; experimental probability
10. No; The mathematical probability is ▤ and the experimental probability is 10/20 or 1/2. When the number of tries is small, what should happen and what actually happens is often different.
11. Answers may vary. There should be more red sections than any other color. One possible solution would be 3 red, 1 yellow, 1 blue, 1 green.

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Name _____

Exploring Two Kinds of Probability

Flip a penny and place a mark to indicate whether it came up heads or tails.
Repeat 10 times for each round.

Round 1

Heads	Tails

Round 2

Heads	Tails

Round 3

Heads	Tails

Total of Rounds 1-3

Heads	Tails

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Name _____

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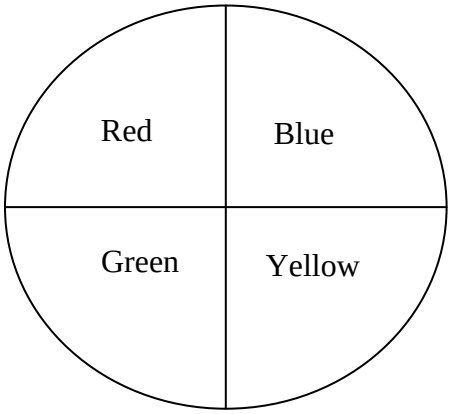


Use the spinner shown at the right to answer the questions.

1. If you spin once, how many different outcomes are possible? What are they?

2. Does each outcome have the same chance? Explain why or why not.

3. If you spin once, what is the probability of spinning blue? What kind of probability is this?

4. If you spin once, what is the probability of spinning red?

5. If you spin once, what is the probability of spinning purple? Explain.

6. If you spin 400 times, about how many times would you expect to get green? Explain.

The table shows the results of Kevin's spins.

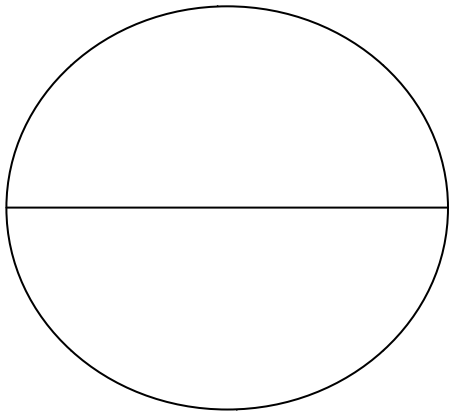
7. How many times did Kevin spin?

8. How many times did Kevin spin blue?

9. Based on the results in the table, what is the probability of spinning blue? What kind of probability is this?

10. Compare your answers for questions 2 and 9. Are the answers the same? Explain why.

11. Label this spinner with color names to create a spinner for which the chances of getting red are greater than the chances for each of the other colors.



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