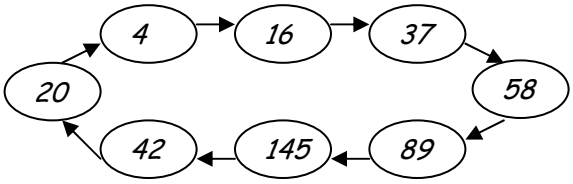
HAPPY NUMBERS

Number theory is a fascinating branch of mathematics which studies the properties of the counting numbers. This activity highlights some interesting facts about a set of numbers called *happy numbers*.

A happy number is a counting number for which the sum of the squares of the digits eventually ends in 1.

Is 19 a happy number?	Is 11 a happy number?
Compute the sum of the squares of the digits of 19. $1^2 + 9^2 = 1 + 81 = 82$ Stop if the result is 1; otherwise repeat the process. Compute the sum of the squares of the digits of 82 $8^2 + 2^2 = 64 + 4 = 68$ $6^2 + 8^2 = 36 + 64 = 100$ $1^2 + 0^2 + 0^2 = 1 + 0 + 0 = 1$ Stop; the result is 1. 19 is a happy number!	$1^2 + 1^2 = 1 + 1 = 2$ $2^2 = 4$ $4^2 = 16$ $1^2 + 6^2 = 1 + 36 = 37$ $3^2 + 7^2 = 9 + 49 = 58$ $5^2 + 8^2 = 25 + 64 = 89$ $8^2 + 9^2 = 64 + 81 = 145$ $1^2 + 4^2 + 5^2 = 1 + 16 + 25 = 42$ $4^2 + 2^2 = 16 + 4 = 20$ $2^2 + 0^2 = 4$ We have returned to 4. The pattern above will repeat indefinitely, never ending in 1. Thus, 11 is not a happy number.

Every number that is not happy will enter this repeating cycle of eight numbers. When a number enters this cycle, it cannot be a happy number because it will never end in 1.



EXERCISES:

1. Twenty of the first 100 counting numbers are happy numbers. Find them; use any shortcuts you discover.
2. Is a happy number plus another happy number always a happy number?
3. Is the product of two happy numbers always a happy number?
4. Is 1776 a happy number year?
5. Find the point in the cycle at which each of these numbers enters.
   (a) 33 (b) 15 (c) 154 (d) 80 (e) 38

[Source: Historical Connections in Mathematics, Volume I ©1992 AIMS Education Foundation, p. 80]