Permission Granted

Exploring Arithmetic Sequences

Objective → Objective → Students will identify arithmetic sequences. They will find missing terms in arithmetic sequences and graph the resulting patterns.

Context → Context → Prior to this lesson, students have had experience describing and continuing various patterns involving whole numbers, decimals, integers, and shapes. They are also familiar with coordinate graphing. They will go on to learn about geometric (common ratio) patterns and other principles of number theory.

NCTM Standards Focus

In this lesson, students explore real-life situations to identify patterns, then determine whether those patterns constitute arithmetic sequences. By representing the relationships found in tables, graphs, and finally algebraic notation, students affirm and expand their understanding of the relationships involved and are able to discern the advantages each form of representation offers. Discussing the various methods used in solving for missing terms in the sequences, then using alternative approaches to verify their results, allows students to select a method that is comfortable for them to use and strengthens their problem-solving abilities.

Reasoning and Proof Students discover defining characteristics of arithmetic sequences by comparing and contrasting various sets of numbers. They use reasoning to find missing terms in arithmetic sequences, and by justifying their generalizations, gain insight into the patterns created.

Representation Students represent arithmetic sequences in tables, algebraically, and with graphs. Using multiple representations helps affirm understanding of the relationships formed and provides students with alternate methods for solving problems.

Communication As students describe sequences verbally, they gain experience with recursive definitions. By sharing their ideas, students experience the key features of arithmetic sequences and reinforce their understanding of how to solve problems involving the patterns formed.

Teaching Plan

Materials → Student pages A.66-67; graph paper

Begin the lesson by presenting a situation to engage students in working with arithmetic sequences.

[Source: Connect to NCTM Standards 2000-Sixth Grade, p. 56-61, Creative Publications, Inc.]

Two radio stations are having listener call-in contests. Each contest begins with a certain amount of prize money, and one caller per hour gets the opportunity to win the money by correctly answering a music question. If the caller does not answer correctly, money is added for the next hour. The tables show the amount of prize money that will be available to the next caller after each incorrect answer.

Radio Station WTMD

Caller Number	**1**	**2**	**3**	**4**	**5**
Prize	**$100**	**$150**	**$200**	**$250**	**$300**

Radio Station WCPB

Caller Number	**1**	**2**	**3**	**4**	**5**
Prize	**$100**	**$125**	**$175**	**$250**	**$350**

Suppose you were the seventh caller to WTMD and you answered correctly. How much money would you win? Explain how you know. ($400; $50 is added to the previous amount for each incorrect answer.) *As the seventh caller answering correctly on WCPB, how much would you win? Explain.* ($625; The amount added increases by $25 for each caller.)

How are the sequences of money formed for WTMD and WCPB the same? (Both start with $100; both are increasing patterns.) *How are the sequences different?* (The amount added for WTMD is the same each time; the amount added for WCPB is different each time.)

EXPLAIN THE TABLE for WTMD demonstrates a pattern called an *arithmetic sequence*, which is formed by adding or subtracting the same number repeatedly. Have students examine the following series of numbers and identify whether or not they are arithmetic sequences, and explain why.

2, 9, 16, 23, 30, …(Yes; 7 is added each time.)

4, 5, 7, 10, 14, …(No; the number added increases by 1 each time.)

2, 4, 8, 16, 32, …(No; the pattern involves multiplication.)

80, 70, 60, 50, 40, … (Yes; 10 is subtracted each time.)

Prompt students to consider how arithmetic sequences could be represented graphically. *How can we show the information in the table for WTMD with a graph?* (Show number of callers on the horizontal axis and money on the vertical axis.) *What do you think the graph will look like? Why?* (Answers will vary. Some students may visualize a set of points that are along a line.)

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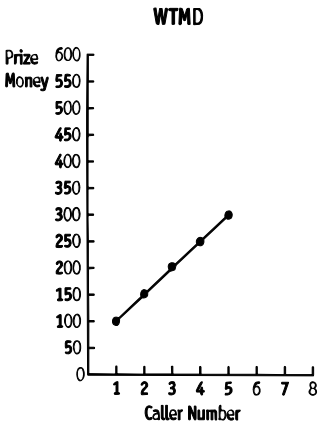
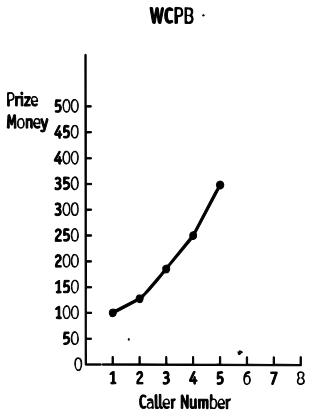


Have students work in pairs to draw a graph of the information for WTMD. Suggest that they try to continue the visual pattern to show other ordered pairs that belong to the sequence. You may also wish to have students draw the graph for WCPB for comparison.

Allow time for students to complete their graphs, then discuss the results. Focus on the graph for WTMD. Point out that the graph of the pattern of any arithmetic sequence will be points lying along a straight line.

Encourage students to develop a formula that expresses the WTMD prize money for any caller by using the beginning amount and the common amount that is being added. Display the following table.

Caller Number	**1**	**2**	**3**	**4**	**5**
Amount Added To $100	**0(50) = $0**	**1(50) = $50**	**2(50) = $100**	**3(50) = $150**	**4(50) = $200**
Total Prize	**$100**	**$150**	**$200**	**$250**	**$300**



What do you notice about the number of times $50 is multiplied to get each prize? (It is one less than the number of callers.) *If n represents the number of callers, what is a formula for the prize money?* $(100 + (n - 1)50)$ Use this formula to find the prize money for the tenth caller. $(100 + 9(50) = 100 + 450 = 550$. The prize is $550.) *How can you check this answer?* (Find the amount by continuing the pattern.)

Summarize this part of the lesson by explaining that for any arithmetic sequence, the *n*th number or term of the pattern can be expressed as $nth\ term = a + (n - 1)d$. In this sentence, n represents the number of the term in the sequence, a represents the first term in the sequence, and d is the common amount that is added or subtracted. Reinforce this formula by reviewing the terms and working through an example with the students.

FINALLY, ASK STUDENTS to use what they know about arithmetic sequences to solve the following problem:

[Source: Connect to NCTM Standards 2000-Sixth Grade, p. 56-61, Creative Publications, Inc.]

The first and fourth numbers in an arithmetic sequence are 7 and 25. Find the rule for this sequence. Find the second and third terms in the sequence.

Pose questions to help students as they think through the process. *Will the rule* than *involve addition or subtraction? How do you know?* (25 is greater than 7, and so the rule must be to add.) *What is the total amount added to 7?* (18) *How many times has the same number been added to 7 to get 25?* (3) *What number is being added each time?* (6) *What are the missing second and third numbers?* (13, 19)

If time allows, have students work in pairs to write one or two problems involving arithmetic sequences. Then have them trade with other students to find the rules and solve the problems.

Student Pages

Page 66 provides practice in identifying arithmetic sequences, writing rules and formulas, and finding missing terms in the sequence. Page 67 allows students to apply their knowledge of arithmetic sequences to solve problems involving real-world settings.

Assessment

As students responded to questions, continued patterns, created graphs, wrote formulas, and found missing terms during the lesson, you could assess their understanding of arithmetic sequences. Student-created problems, as well as their work on the exercises on the student pages, also provided opportunities to evaluate students' fluency with the concepts.

NCTM Standards Summary

Representation was emphasized in this lesson as students used tables, graphs, and algebraic symbolism to express arithmetic sequences. This approach helped students to understand how mathematics provides several methods for working with the same relationship. By applying their own reasoning processes to the representations, students developed the important ideas for themselves. Throughout the lesson, students discussed their work, clarifying ideas for themselves and their audience of peers.

[Source: Connect to NCTM Standards 2000-Sixth Grade, p. 56-61, Creative Publications, Inc.]

Answers

Page A.66

1. Yes; add 4 each time
2. Yes; subtract 8 each time
3. No; multiply by 3
4. Yes; add 0.2 each time
5. No; add 3, then 4, then 5, …
6. Add 8; 27, 35, 43
7. Subtract 3; 36, 33, 30
8. Add 2; 8.3, 10.3, 12.3
9. Subtract 0.25; 4.25, 4, 3.75
10. Add 1.4; 36.2, 37.6, 39.0
11. 3; nth term = $3 + (n - 1)3$; 10th term = 30
12. 2; nth term = $2 + (n - 1)5$; 10th term = 47
13. 15, 24
14. 19, 30
15. 37, 32

Page A.67

1. Yes; the rule is add 4.
2. 24, 28, 32
3. nth perimeter = $4 + (n - 1)4$
4. 15th perimeter = $4 + 14(4) = 60$
5. nth perimeter = $4n = 4 + (n - 1)4 = 4 + 4n - 4 = 4n$
6. Answers will vary.
7. 44 wpm; 12th day
8. 7; 25, 32, 39

Name _____

Exploring Arithmetic Sequences

Is the pattern an arithmetic sequence? Write *yes* or *no*. Explain.

1. 0, 4, 8, 12, …
2. 100, 92, 84, 76, …
3. 1, 3, 9, 27, …
4. 2.8, 3.0, 3.2, 3.4, …
5. 3, 6, 10, 15, …

The first three terms of an arithmetic sequence are given. For each sequence, describe the rule and write the next three terms.

6. 3, 11, 19, …
7. 45, 42, 39, …
8. 2.3, 4.3, 6.3, …
9. 5, 4.75, 4.50, …
10. 32, 33.4, 34.8, …

For each arithmetic sequence given below, (a) identify the first term; (b) write a formula for the *n*th term; (c) use the formula to find the tenth term in each sequence; (d) check your answer.

11. 3, 6, 9, …
12. 2, 7, 12, …

Find the second and third terms of the arithmetic sequence if

13. the first number is 6 and the fourth number is 33.
14. the first number is 8 and the fourth number is 41
15. the first number is 42 and the fourth number is 27.

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Name _____

Exploring Arithmetic Sequences

The table shows the length of a side of a square garden and the perimeter of the garden.

Length of Side	1	2	3	4	5
Perimeter	4	8	12	16	20

1. Do the numbers in the perimeter row form an arithmetic sequence? Explain.

2. What would be the next three numbers in the perimeter row?

3. Write a formula for the perimeter of a square of side *n* (the *n*th perimeter) using the first number (perimeter) in the pattern.

4. Use your formula to find the 15th perimeter in the pattern.

5. Do you know another formula for the perimeter of a square of side *n*? Write this formula. Explain why this formula and the one you wrote above give the same perimeters for any value of *n*. (Hint: Use the distributive property.)

6. Draw a graph showing the relationship between the length of the side of a square and its perimeter. Describe your graph.

Solve each problem.

7. Laurie typed 24 words per minute the first day she practiced, 28 words per minute the second day, and 32 words per minute the third day. If the pattern continues, what will Laurie's typing speed be on the sixth day? On which day will Laurie's typing speed be greater than 65 words per minute?

8. Roberto began his collection of trading cards with 18 cards that were a present from his grandfather. Each week, he added the same number of cards to his collection. At the end of five weeks, he had 46 cards. How many cards did Roberto add each week? How many cards did he have at the end of the second, third, and fourth weeks?

[Source: Connect to NCTM Standards 2000-Sixth Grade, p. 56-61, Creative Publications, Inc.]