Understanding Prime and Composite

Introduction

Objective → Students will understand prime and composite numbers.

Context → Students have learned the difference between odd and even numbers. They will use what they have learned to work with prime factorizations, greatest common factors, and least common multiples.

NCTM Standards Focus

Most lessons that introduce prime and composite numbers do so by simply defining the two terms and having students classify certain numbers. For students who do not yet have a solid understanding of factors, a lesson of this nature may be difficult. A more standards-based lesson would develop the concepts in greater depth and with a visual representation, rather than with just an abstract definition. Incorporating the following three process standards into the lesson will help students better understand prime and composite numbers.

Reasoning and Proof Students separate numbers into two types: prime and composite. They use visual representation of the two types of numbers to confirm that their conclusions are correct.

Representation Students use square tiles to create visual representations of prime and composite numbers. They form rectangles to represent the factors of a number. Then they separate numbers into primes and composites based on whether only one rectangle or multiple rectangles can be formed to represent a given number.

Connections Once students have separated numbers into the two types using visual representation, they can make connections to the terms prime and composite. Students use their knowledge of multiplication and division to make connections to prime and composite numbers.

TEACHING PLAN

Materials → Student pages A.6-7; 465 square tiles; chart on page A.3

DISPLAY A RECTANGULAR ARRANGEMENT of 15 tiles and ask the class how many tiles are used. Ask how they know. When the response is that there are 3 rows of 5 tiles, ask what multiplication problem this represents. Point out that one rectangle represents 3x 5 and 5 x 3. Remind students that 3 and 5 are factors of 15.

Write the chart shown on page A.3 on the board or overhead. Have students work together in pairs or groups. Give one group 10 tiles and assign them the numbers 2 through 10. Tell them that for each number, they must use that many

[Source: Connect to NCTM Standards Sixth Grade, p. 24-29, Creative Publications, Inc.]

tiles to make their rectangles. For example, if they are working with the number 6, they must find as many ways as possible to create rectangles using 6 tiles. Give the next group 15 tiles and ask them to use the tiles to make as many rectangles as possible for each number from 11 through 15. Continue in this manner so that groups of students work with all of the numbers on the chart.

This activity will provide students with a visual representation of prime and composite numbers. It will also help students separate numbers into those that have exactly two factors and those that have more than two factors.

USING THE RECORDING SHEET on student page A.6, have students record the number of tiles they started with, the dimensions of each rectangle created, the total number of rectangles created for each number, and the total number of factors for that number.

Number of Tiles	Make Rectangles for These Numbers
10	2-10
15	11-15
20	16-20
24	21-24
28	25-28
32	29-32
36	33-36
40	37-40
44	41-44
48	45-48
52	49-52
56	53-56
60	57-60

What Might Happen…What to Do

There are two common errors that students might make during the activity. First, they might count 3 x 5 and 5 x 3 as two different rectangles. If so, they could list 3 and 5 as factors twice. Second, they might not realize that a square is a rectangle. As a result, they might consider some square numbers as prime. If you observe students making either of these errors, discuss them with the group or the class.

Each group of students should examine their data and write down their thoughts and observations about their findings before they share them with the class. Begin class discussion by compiling a list on the board showing the numbers 1-60, the total number of rectangles made for each number, and the total number of factors. Have students look back through their written thoughts and observations to see whether they are true for all of the numbers. This will help students connect what they noticed about their set of numbers to all of the numbers.

It is important that students focus on the goal of this lesson. *How many factors do the numbers with one rectangle have? What do you notice about the numbers that have exactly two factors?* (All are odd except for two.)

[Source: Connect to NCTM Standards Sixth Grade, p. 24-29, Creative Publications, Inc.]

ASK THE STUDENTS TO SHARE their thoughts with the class and to come up with one or more general statements about the numbers. Students are using reasoning and proof at several levels when they connect their initial thoughts and observations with the results of the rest of the class and then make generalizing statements.

Students may come up with one or more of the following observations:

- All of the numbers that have only one rectangle have two factors, except 1.
- The numbers that have only two factors have only one possible rectangle.
- Two is the only even number that has a total of 2 factors.
- All other even numbers have more than one rectangle.

Now make the connections to the important terms of the lesson. Define PRIME numbers as those numbers that have exactly two factors. *What are the two factors of a prime number?* (The number itself and 1) Define COMPOSITE numbers as those numbers that have more than two factors. A good question to ask to help students realize the distinction is: *Can a number be both prime and composite?* (No) *Why not?* (A prime number has only two factors; a composite number has more than two factors) Give students time to ask any questions they may have.

ASK STUDENTS HOW THEY COULD use the hundred chart on student page A.6 to figure out which numbers are prime. If necessary, suggest that they think about the multiples of numbers. Following this discussion, introduce students to the Sieve of Eratosthenes. Since 1 is not prime, students can start with 2. They circle 2, then cross out all the multiples of 2. They circle the next number that is not crossed out, 3, and cross out all the multiples of 3. Students circle the next number that is not crossed out, 5, and cross out the multiples of 5. They continue in this fashion until all the numbers have been circled or crossed out. The circled numbers are the prime numbers. Students can compare this list with the list from their earlier activity. *Can you verify the generalizations you made in the earlier activity with the numbers 61 through 100?*

At this point, you might want to reinforce aspects of the lesson. *Why is 2 the only even prime number?* (All other even numbers have 2 and half the number as factors in addition to 1 and the number itself.) *Should 1 be considered a prime number?* (No, it has only one factor—itself.)

[Source: Connect to NCTM Standards Sixth Grade, p. 24-29, Creative Publications, Inc.]

Extension

Point out that any even composite number can be expressed as the sum of two prime numbers. Have students check to see if this is true for all of the even composite numbers up to 100. For example, the composite number 32 can be expressed as the sum of the two primes, 3 + 29, or 13 + 19.

Student Pages

Student page A.6 features a recording chart for students to use with the activity, as well as a hundred chart for students to use for the Sieve of Eratosthenes activity.

Student page A.7 provides some practice with the concepts of this lesson. To help assess the depth of their understanding, students are asked both to identify a number as prime or composite and to explain what makes it prime or composite.

Assessment

During the activity, you observed students as they made their rectangles and wrote their observations. During the class discussion, you were able to see if students made the connection between the number of factors a number has and whether it is prime or composite. The suggested questions also provided good opportunities to assess students' understanding of the concept of prime and composite numbers. The exercises on student page A.7 provide some questions with which to assess students individually.

NCTM Standards Summary

This lesson focused on the NCTM process standards reasoning and proof, representation, and connections. Students made observations, refined their thoughts, and then made generalizations based on class results. This helped students use their reasoning skills and reinforced the importance of being able to explain their reasoning.

Through using tiles to create visual representations of prime and composite numbers, students had the chance to define these numbers for themselves and to make connections between each term and the concept of factors. The exercises reinforced these connections by asking students to relate each term back to the activity and to connect their description of a number, prime or composite, with the number of factors it has.

Answers

Page A.6

Answers may vary.

Page A.7

1. composite; sample answer: has more than two factors
2. composite; sample answer: can make more than one rectangle
3. prime; sample answer: has only two factors, 1 and 17
4. composite; sample answer: has more than two factors
5. prime; sample answer: can make only one rectangle
6. prime; sample answer: has only two factors
7. neither; sample answer: 1 has only one factor; primes have two; and composites have more than two
8. Only one rectangle is possible for prime numbers; two or more rectangles are possible for composite numbers; examples will vary.

[Source: Connect to NCTM Standards Sixth Grade, p. 24-29, Creative Publications, Inc.]

Name _____

Understanding Prime and Composite

Record your findings about the numbers.

Recording Chart

Number of Tiles	Dimensions of Rectangles	Total Number of Rectangles	Total Number of Factors

Use the hundred chart to find prime numbers.

Hundred Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

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Name _____

Understanding Prime and Composite

For numbers 1-6, determine whether each number is prime or composite.
Give a reason for your answer.

∂ 36

• 45

÷ 17

≠ 93

≡ 79

≈ 2

… Is 1 considered to be a prime number, a composite number, or neither? Explain.

│ Explain how making rectangles from square tiles can help you define and identify prime numbers and composite numbers. Show an example to support your explanation.

[Source: Connect to NCTM Standards Sixth Grade, p. 24-29, Creative Publications, Inc.]