Using Circles

Introduction

Objective → Students will solve problems involving circles and suggest related problems to explore.

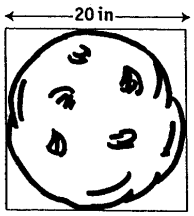
Context → This lesson comes after students have learned to find perimeter and area for quadrilaterals, triangles, and circles. Students have worked with fractions, ratios, and percents. They will go on to study volume and surface area of solid figures.

NCTM Standards Focus
In this lesson, problems involving circles and other figures require students to make a plan and organize the information needed to solve the problem. Students will discover that solving one problem raises other interesting questions that can lead to different paths of exploration. Students will participate in a problem-solving experience that reflects the true nature of mathematics as they look for patterns in their work and suggest alternative situations.

Problem Solving Students break down a complex problem into smaller parts and decide how to work with each part. They consider alternate strategies and evaluate the effectiveness of each. As students focus on the meaning of their solution, they recognize other related problems.

Reasoning and Proof As students progress from the initial problem to more general questions, they make predictions about expected relationships and give reasons for their ideas. Students identify patterns in their work and explain the significance of these patterns. They formulate other problems to test their insights and think about related situations.

Communication Throughout the lesson, students are encouraged to share ideas and create a dynamic problem-solving environment. Students engage in discussions that focus on inquiry rather than getting the right answer. They communicate their insights and understandings verbally and in writing.



Teaching Plan

Materials → Student pages A.37-38

INTRODUCE THE LESSON BY PRESENTING a problem-solving situation for students to consider.

> **Ramon works at the Cookie Connection. The bakery's Monster Chip cookie is packaged in a box with a square bottom that measures 20 inches on a side. As Ramon looked at the cookie sitting in its box, he wondered what percent of the square bottom of the box the cookie covered.**

[Source: Connect to NCTM Standards Sixth Grade, p. 108-113, Creative Publications]

Provide each student with a copy of student page A.37 where they will find the situation represented. Emphasize that the cookie is a circle that exactly fits the box with the sides just touching. *What plan can be used to solve this problem?* Students should suggest these steps for finding a solution.

- Find the area of the square.
- Find the area of the circle.
- Form the ratio (*area of circle/area of square*) and express it as a percent.

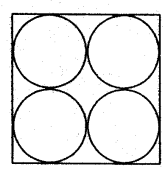
Guide students through the steps to follow to solve the problem.

- *Do we have enough information to carry out each step?*
- *How will you find the area of the square?* (S^2; the length of a side is given.)
- *What is the area of the square?* (400 sq. in.; 20 x 20) Record this area next to the corresponding step on the board.
- *What do you need to know to find the area of the circle?* (The length of the radius)
- *What is the length of the radius? How can we find it?* (10 inches; Students should explain that the diameter of the circle is equal to the length of the side of the square, so the length of the radius is 20 ÷ 2 = 10 in.)

Have students work individually or in pairs to find the area of the circle using 3.14 for π. (A = 314 sq. in.; 3.14 x 10^2) Then have them complete the problem by finding the percent. (314 ÷ 400 = 0.785; 0.785 x 100 = 78.5%.)

Present a variation of the problem for students to consider.

Four Big Chip cookies fit in the same box as the Monster Chip. Ramon wonders what percent of the square bottom the 4 smaller cookies will cover.



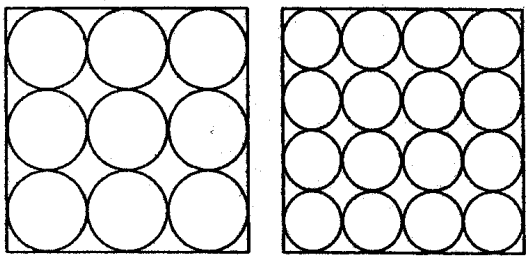
Invite students to predict how they think the percent of the square covered by the 4 cookies will compare with the answer they found for the Monster Chip and to explain their thinking. Encourage students to give their ideas until all three

[Source: Connect to NCTM Standards Sixth Grade, p. 108-113, Creative Publications]

possibilities, less than, greater than, or equal to, have been expressed. Whether students' predictions are correct is not important since the results of the problem are counter-intuitive; the goal is to encourage students to engage in mathematical reasoning. Students may use imprecise terminology such as "little bits of area," but their arguments should reflect a logical structure.

Have students suggest a plan to solve the question. They should suggest finding the area of one small circle, multiplying that by 4 to get the total area, and then using the area of the square that they already know to compare the areas and express the comparison as a percent. Have students elaborate on how they will determine the length of the radius of the smaller circle. They should articulate that the diameter of each circle is half the side of the square and the radius is half of the diameter, or 5 in. *What is the total area covered by the cookies?* (314 sq. in.) At this point, observant students will note that since the total area is the same as for the Monster Cookie, they know the percent without any further computation—again, the answer is 78.5%.

Ask students if this result surprised them. *Based on these two problems, what question might occur to you?* Students may wonder if the area covered will always be 78.5% no matter how many circles are used. *What are the next two figures in this pattern?* (9 circles and 16 circles) Encourage students to investigate on their own. Instruct them to draw sketches. Using a calculator will facilitate computation. Students will find the circular areas continue to be 78.5% and they may find it interesting to speculate why.



Prompt students to consider a related problem.

The square used in the above problems is 20 inches on a side. If the dimensions of the square were changed to, say 50 inches or 100 centimeters, would the percentage change?

Answers

Page A.37

1. 400 in.^{2}; 314 in.^{2}; 78.5%
2. 400 in.^{2}; 314 in.^{2}; 78.5%
3. Square: A = 400 in.^{2}
   Circles: 9 circles; 314 in.^{2}; 78.5%; 16 circles; 314 in.^{2}; 78.5%
4. Answers may vary. When any number of congruent circles are drawn (inscribed) inside a given square so that each circle touches the circle next to it and the sides of the circles touch the sides of the square, the area covered by the circles is always 78.5% of the area of the square.

[Source: Connect to NCTM Standards Sixth Grade, p. 108-113, Creative Publications]

Students could measure the areas again or they could use reasoning. Changing the dimensions would not change the ratio of the radius to side length. The percent of the circular area would be the same; 78.5%.

Have students consider a different situation.

> **Suppose a square cake was packed, *inscribed*, in a circular tin of given diameter. What percent of the circular bottom would the cake cover?**

Although an analysis of this latter situation is beyond the students' skills at this time, the goal of the discussion is to have students recognize how problem solving in mathematics grows from one situation into many others. As students continue to learn about three-dimensional figures, they may formulate analogous packing problems using volumes.

If time allows, you may wish to have students begin work on some of the other problems on the student pages and discuss any questions that arise.

Student Pages

Student page A.37 provides figures and recording space for the problems discussed during the lesson. Student page A.38 includes additional problems involving measurements of circular regions and offers the students an opportunity to create their own problems.

Assessment

As students responded to questions and solved problems, it was possible to assess their ability to make a problem-solving plan and find needed information. As they worked, you were able to evaluate their proficiency at finding areas and carrying out other computations. The discussion about related problems provided an opportunity for you to note those students who could think creatively about mathematics.

NCTM Standards Summary

The problem-solving situations in this lesson served as a springboard for new questions. Students used reasoning to identify patterns for further investigation, to make predictions about expected results, and to justify their beliefs. They were encouraged to discuss alternate strategies and describe related problems that interested them. The approach of this lesson encouraged students to express their understandings and insights and helped them to recognize the ongoing nature of mathematical problem solving.

[Source: Connect to NCTM Standards Sixth Grade, p. 108-113, Creative Publications]

Answers

Page A.38

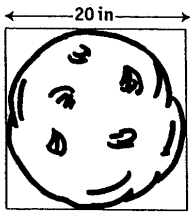
1. 628 sq. mm
2. 4 in.
3. 16 in.^{2}
4. 4π
5. ▤ of the area
6. Subtract 4 x ▤ (area of cookie) from square.
7. $16 - 4\pi \approx 3.44$ in.^{2}
8. One method is to find area of square, find area of one whole circle (since 4 x ▤ = 1); subtract to find good land, compare good land to area of square and determine if it is greater or less than half.
9. Area (square) = 10,000 sq. meters; Area (circle) = $1600\pi \approx 5{,}024$ sq. meters; Swamp is greater by 48 sq. m.
10. Answers may vary.

Name _____

Using Circles

Solve each problem.

∂ What percent of the square does a Monster Cookie cover?
Assume the cookie is a circle.

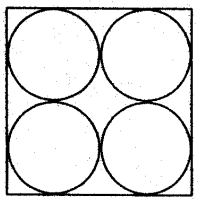


Area of square _____

Area of circle _____

Percent _____

- What percent of the square do the four smaller cookies cover?



Area of square _____

Area of circle _____

Percent _____

÷ Draw the next two figures in the pattern. Find what percent of the square is covered by each.

Area of square _____

Area of circle _____

Percent _____

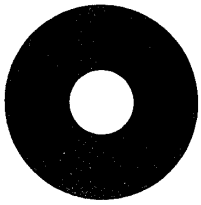
≠ Write a statement summarizing your findings.

[Source: Connect to NCTM Standards Sixth Grade, p. 108-113, Creative Publications]

Name _____

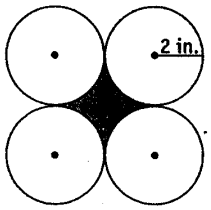
Using Circles

Solve each problem. Use $\pi = 3.14$.



∂ What is the area of the metal washer if the radius to the outer circle is 15 millimeters and the radius of the hole is 5 millimeters?

Four L′il Chip cookies are placed together as shown at the right. Draw a square that has one corner in the center of each cookie.



- Find the length of the side of the square.

÷ Find the area of the square.

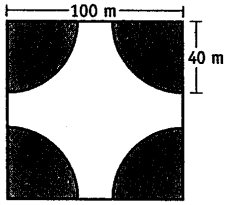
≠ Find the area of each cookie. Express your answer in terms of π.

≡ What part of the area of each cookie is contained in the square you drew?

≈ Explain the method you used.

… What is the area of the shaded region?

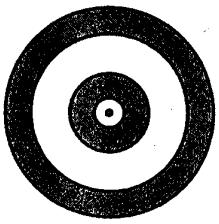
Mr. Coole bought the square piece of land shown. Unfortunately, he discovered the four shaded regions (four quarter circles) were swamps and not much good for anything.



| Which does he have more of—swamp or land? How much more?

— Explain the steps you used to solve the problem and how you found the information you needed.

↵ Create some problems suggested by the target shown.



[Source: Connect to NCTM Standards Sixth Grade, p. 108-113, Creative Publications]