Task 6

Overview

Reason about probability
Justify a decision
Communicate mathematical reasoning

Lucky Draw

Long Task

Task Description

Students use theoretical and/or experimental probability to determine the profitability of a festival game. They write a report to the festival committee explaining their recommendation.

Assumed Mathematical Background

It is assumed that students have had experience with finding theoretical and experimental probability.

Core Elements of Performance

- analyze and reason about probability to determine the profitability of a carnival game
- make and justify a recommendation to an audience

Circumstances

Grouping:	Following a class introduction, students complete an individual written response.
Materials:	red and blue colored counters or cubes, 3 large cups (for class demonstration), and additional cups for students
Estimated time:	40 minutes

Name Date

Lucky Draw

This problem gives you the chance to

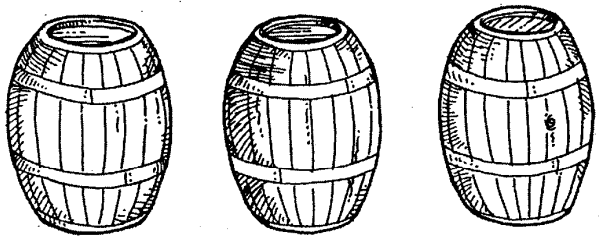
- *analyze and reason about probability*
- *make and justify a decision*
- *communicate mathematical reasoning*

At the Palatine School's fall festival, the Charity for Children Club is planning to run a money-raising booth. One of the members in the club proposed the following game:

Lucky Draw

Win $1.00

10¢ per turn



There are equal numbers of red and blue balls buried in sawdust in each barrel.

One turn allows you to make
ONE LUCKY DRAW from each barrel for 10¢.
If you draw three balls of the same color on one turn you win $1.00.

Elida, the chairperson of the festival, likes the idea of the game, but she wants to make sure it is a good money maker. As co-chairperson, you have been asked to prepare a report to the festival committee on this issue. Make sure that your report includes your recommendation and clearly explains how you came to your conclusion.

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A Sample Solution

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Theoretical Approach A

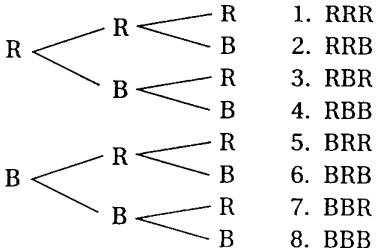
I believe that this game is not a good fund raiser. I figured out the following about the game:

- An equal number of red balls and blue balls means that there is a $\frac{1}{2}$ probability of drawing either red or blue from each barrel.
- If I want to draw three red balls (one from each barrel), then the probabilities must be multiplied: $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$. This is the same for three blue balls.
- So the probability of getting three balls of the same color is $\frac{1}{8}$ for red and $\frac{1}{8}$ for blue. So, together there is a $\frac{1}{4}$ probability of winning the \$1.00 prize.
- That means that for every four players, one of them should win. The game would take in \$0.40 for every four players and pay out \$1.00, leaving the charity \$0.60 behind. So they could expect to lose \$0.60 ÷ 4 players or \$0.15 per player on average when a lot of people play this game.

If a lot of people were to play this game, a lot of money could be lost and there would be no profit at all. I would not recommend this game to Elida as a "money maker."

Theoretical Approach B

To find out if the game would be profitable, I made a tree diagram. *R* is a red ball. *B* is a blue ball.



The diagram shows that there are eight different ways of drawing red and blue balls from the three barrels. Two of them, RRR and BBB, give the player a win. So there is a $\frac{2}{8}$ or $\frac{1}{4}$ chance of winning. So for every eight plays, the charity would take in \$0.80 and pay out \$2.00, which gives them a loss of

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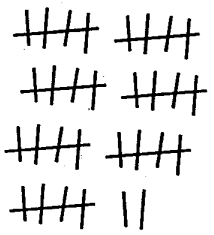
$1.20. If a lot of people played this game, they would lose a lot of money. The game needs to be changed.

Experimental Approach

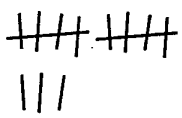
I played this game by putting an equal number of red and blue chips in three cups and then drew a chip from each cup 50 times (without looking). I put the chips back in their cups and mixed the chips very well after each play. Here are my results for the game:

Losses



37 losses
Charity club makes $3.70
(Each player paid $0.10,
so 37 × $0.10 = $3.70.)

Wins



13 wins
Charity club pays $11.70
(Each player paid $0.10, but the charity club paid out $1.00 to each of them,
so 13 × $1.00 = $13.00 – $1.30 = $11.70.)

Overall in this game, experimental probability shows that for every 50 players, we would make $3.70 on the losses and pay out $11.70 on the wins, giving a total loss of $8.00. I think I played this game enough times to show what will happen when a lot of people play it. I would expect that the charity will lose a lot of money with this game. It needs to be changed.

Using this Task

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For Formal Assessment

Hand out the task to students and read the aims of the assessment in the box at the top of the activity page. Read through the task with students and make sure they understand the context of the task. You may want to model the game with colored cubes or counters and cups. Have such materials available for students to use while completing the task. Answer any questions about the *context* of the task, but if students ask what form of probability they should use (experimental or theoretical), tell them it is their choice.

Issues for Classroom Use

Students may choose to use either theoretical or experimental probability to determine whether or not the game is profitable. If students use only experimental probability, a complete response must include an explanation of how the game was modeled (for example a student could draw three times from only one cup, replacing each ball after each draw) and must involve a large number of trials (50 or more). If students use *both* theoretical and experimental probability, a high-level response would include some discussion about the differences in the findings, the meanings of the two findings, and a reconciliation between the two in the recommendations to the festival committee.

This task may be extended in numerous ways. If many students use experimental probability, have them share the different findings and discuss why they may differ from each other and from the theoretical findings. Pose or have students pose different questions or change the parameters of the game (for example, what if there were four barrels to draw from, or n barrels? What if there were three different colors of balls? What if there were twice as many red balls as blue balls? What if we didn't know whether there were equal numbers of each color ball in each barrel?). Alternately, have students play with the different parameters—including cost to play and cost of prize—and have them design a profitable game.