

GRADE EIGHT – CONTENT STANDARD #7
EXTENDED LESSON A

Activity 8

NEWSPAPER PAY

OVERVIEW

This is a summary activity for several themes that have previously been studied. The following problem is posed. A newspaper carrier is offered a choice by a customer of two pay plans: the usual \$5 a week, or the money received from drawing two bills out of a sack containing one \$10 bill and five \$1 bills. Two sums are possible: \$11 or \$2.

Students are given an opportunity to plan how to simulate the problem, to carry out the simulation, to analyze the problem theoretically, and to compare the results. The notions of expected value and fair games are combined in a more formal way, allowing students to review and reinforce both ideas. For example, the expected value (money) of drawing two bills from a sack is

$$P(\$11) = \frac{5}{15} = \frac{1}{3} \qquad P(\$2) = \frac{10}{15} = \frac{2}{3}$$

To find the expected value, we look at what happens in the long run. Since we have 3 in the probability denominators, let's look at 30 trials.

The number of times we expect the newspaper carrier to get \$11 is $\frac{1}{3} \times 30$, or 10, times.

She will get \$2 about $\frac{2}{3} \times 30$, or 20 times.

Over the 30 weeks she will receive: $10 \times \$11 + 20 \times \$2 = \$150$.

This is an average (or expected value) of $\frac{\$150}{30} = \5 per week.

So, if this pay plan is continued over many weeks this is a fair deal. The newspaper carrier should collect about the same under both plans.

The practice problems investigate some pay plans that are not fair over the long run. They also require reviewing many analyses that the students have done earlier, such as analyzing the outcome for tossing three coins or two dice.

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Goals for students

1. Understand simulation by gathering, organizing, and analyzing data.
2. Devise a simulation plan.
3. Determine the expected value or long-term average.
4. Make organized lists of possible events.
5. Determine if games are fair or unfair.

Materials

Each group needs *one* of the following sets of items:
5 Ping Pong balls marked 1; 1 Ping Pong ball marked 10; 1 large sack.
Bobby pins for spinners and spinner bases.
6 stiff cards and 1 large sack.
One \$10 bill and five \$1 bills in play money & 1 large sack.

Worksheets

- *8-1, The Newspaper Offer
- 8-2, Newspaper Pay

Transparencies

Starred item should be made into a transparency.

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| TEACHER ACTION | TEACHER TALK | EXPECTED RESPONSE |
|---|--|--|
| Tell a story. | Sue delivers newspapers for \$5 a week per customer. One customer makes the following suggestion. Instead of my paying you \$5 each week, let's make it more interesting. Each week I'll put five one-dollar bills and one ten-dollar bill in a paper sack. You'll reach in, without looking, and take two bills. You might only get \$2, but you <i>might</i> get \$11. How about it? Think it over tonight and give me your answer tomorrow. | |
| Ask. Record opinions. | Does this sound like a good deal to you? If you were Sue what would you do to help you make the decision? | Various answers. Simulate the drawing. |
| Place six Ping Pong balls in a sack. Have a volunteer draw two balls. Repeat five additional times. Each time, be sure to return the balls to the sack. | Let's simulate the drawing by using Ping Pong balls placed in a sack. One ball is marked 10 and five are marked 1. | |
| Record the number of \$11 and \$2 outcomes. | Did Sue come out ahead for the first six weeks? | $(\$2 \times \square) + (\$11 \times \square) = \$\square$ |
| Explain. | Six simulations may be too few to give us a true picture of Sue's chances. Let's simulate the drawing 30 times in each of the groups. The first thing you must do in your group is to plan a way to simulate the problem so that the data you gather will correctly represent Sue's possibilities. You have seen one way to do this using Ping Pong balls. You know that we have spinners, cards, marbles, discs, dice, coins, and various other items to use. | |
| Pass out Worksheet 8-1, The Newspaper Offer. | | |

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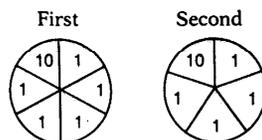
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TEACHER ACTION**TEACHER TALK****EXPECTED RESPONSE**

Give the groups a few minutes to try to devise a simulation plan. If needed, give students a bag containing equipment that will allow one of the following methods, but put in too much equipment so that students still have to make some decisions.

The following are some reasonable ways to simulate the problem.

Use two spinners, one marked in six equal parts and one marked in five equal parts.



If the spin on the first spinner hits the 10, record an \$11 outcome. If the spin on the first spinner hits a 1, spin again on the second and record \$11 or \$2 depending on what is hit.

Use six appropriately marked Ping Pong balls in a container. Six marbles or six discs could also be used.

Use six cards marked and shuffled (old playing cards can be marked and used).

Use paper money, one ten-dollar bill and five one-dollar bills in a sack (this requires *care* to see that the money is shuffled well each time).

(Be careful of dice. They do not allow the 5 options for the second throw.)

A.159

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| OBSERVATIONS | POSSIBLE RESPONSES |
|--------------|---|
| | <p>Check each group to be sure they are using a correct simulation scheme. If a group finishes early, have them simulate in a different way and report on the comparative results. Did they feel one way was more random than the other, easier and faster to get good results, etc.?</p> <p>Ask students to describe ways to make sure that they are not influencing the outcomes. Elicit such responses as spin on a flat surface, etc.</p> |

A.160

| TEACHER ACTION | TEACHER TALK | EXPECTED RESPONSE |
|----------------|---|-------------------|
| Ask. | Suppose Sue collected \$5 every week for 30 weeks. How much is that? | \$150. |
| Ask. | <p>How many groups got more than \$150?</p> <p>How many got less than \$150?</p> <p>Did anybody get \$150?</p> <p>What's the most any group got?</p> <p>What's the least any group got?</p> | Various answers. |

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TEACHER ACTION

Display a transparency of Worksheet 8-1. Record each group's results in a table.

Example:

| Group | No. of \$11 | No. of \$2 | Total Money |
|--------|-------------|------------|-------------|
| 1 | 9 | 21 | \$141 |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |
| 6 | | | |
| ⋮ | | | |
| Totals | | | |

Recording the class data on each student's sheet makes a permanent record that can be used to keep students who miss class up to date.

TEACHER TALK

Let's record each group's data so that we can make a whole-class summary.

How would we find the *average* amount of money that each group totaled?

Let's do the calculations. What do we get?

If we divide this number by 30, we have the class average per trial. What is it?

As a result of our class trials do you think this is a fair deal for Sue over the long run?

EXPECTED RESPONSE

Add all the money and divide by the number of groups.

Approximately \$150.

Approximately \$5.

Various answers.

A.161

We are going to analyze carefully what Sue can expect. Suppose these are the bills in the sack. We'll mark the ones 0₁, 0₂, 0₃, 0₄, 0₅, and the ten will be T.

Tell me one combination of two bills.

Various answers. (For a response "ten and one," ask "which one-dollar bill?")

Have students list combinations of bills worth \$11 and worth \$2.

- T,0₁ 0₁,0₂ 0₂,0₃ 0₃,0₄ 0₄,0₅
- T,0₂ 0₁,0₃ 0₂,0₄ 0₃,0₅
- T,0₃ 0₁,0₄ 0₂,0₅
- T,0₄ 0₁,0₅
- T,0₅

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| TEACHER ACTION | TEACHER TALK | EXPECTED RESPONSE |
|--|---|--|
| Some students may suggest using a tree diagram. This will give every outcome in both orders, such as T ₀ and O ₁ T. The probabilities remain the same. | | |
| Ask. | How many combinations are worth \$11? | Five |
| | How many combinations are worth \$2? | Ten |
| | What fraction (probability) of the ways is worth \$11? | $P(\$11) = \frac{5}{15} = \frac{1}{3}$ |
| | What fraction (probability) is worth \$2? | $P(\$2) = \frac{10}{15} = \frac{2}{3}$ |
| | One-third of the time, Sue expects \$11. | |
| | Two-thirds of the time, Sue expects \$2. | |
| Ask. | What would we expect the long-term average to be? | Various answers. |
| | Is this the same as the <i>expected value</i> of each draw? | Yes. |
| | In 30 draws, how many times will Sue expect to get \$11? How many times will she expect to get \$2? | Ten times 20 times |
| | How much money would this total? $\$11 \times 10 + \$2 \times 20 = \square$ | \$150 |
| | Divide by 30 to get the average per draw. What is the average? | \$5 |
| Ask. | Is this a fair deal over the long run? | YES! |
| | Would you take the chance? | Various answers. |
| Pass out Worksheet 8-2, Newspaper Pay. | Here are some other situations for you to analyze. Decide which ones are fair. | |
| Do problems 1, 3, and 5 in class to make sure everyone has the correct procedures and understandings. | | |

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NAME _____

The Newspaper Offer

1. Describe how you will conduct the experiment. What are you using to represent the money? How are you drawing out the money? Is the way you are collecting the money random?

Record your data.

| |
|--|
| Mark a tally in this box when Sue gets \$11. |
| Money collected: Total × \$11 = _____ |

| |
|---|
| Mark a tally in this box when Sue gets \$2. |
| Money collected: Total × \$2 = _____ |

Total number of trials = _____

Total money collected = _____

Long term average = $\frac{\text{Total money collected}}{\text{Total number of trials}}$ = _____

2. Class Data:

| | | | | | | | | | | | | | | | | | | | |
|-----|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|
| 11s | | | | | | | | | | | | | | | | | | | |
| 2s | | | | | | | | | | | | | | | | | | | |

Total 11s = _____

Total 2s = _____

Total trials = _____

Total money collected = _____ × \$11 + _____ × \$2 = _____

Long term average = $\frac{\text{Total collected}}{\text{Total trials}}$ = _____

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Worksheet 8-1

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NAME _____

Newspaper Pay

In each of the following situations, the customer should pay \$5 per week for newspapers. Sue, the paper carrier, has to decide which of the schemes of chance would give her a fair deal over the long run. In each case, decide what Sue should do—accept or reject the proposal.

1. The customer will place a five-dollar bill and 3 one-dollar bills in a bag. Sue will draw out two bills.
2. The customer will place a five-dollar bill and 2 one-dollar bills in a bag. Sue will draw out two bills.
3. Sue will toss three coins. If two or more land heads, Sue gets \$9. Otherwise, she gets \$1.
4. Sue will toss three coins. If all land the same, all heads or all tails, Sue gets \$15. Otherwise, she gets \$1.
5. Sue rolls a pair of dice. If the sum is exactly 7, Sue gets \$20. Otherwise she gets \$2.
6. Sue will roll a pair of dice. If the sum is at most 4, Sue will get \$20. What should she get otherwise to make the payoff fair in the long run?

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Worksheet 8-2

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