

Name \_\_\_\_\_

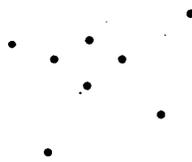
Date \_\_\_\_\_

# Compact-Ness

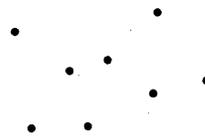
This problem gives you the chance to

- *define a measure of "compact-ness"*
- *evaluate the measure*
- *explore the range of applicability of the measure*

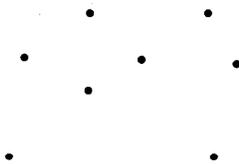
Below are four clusters of eight dots each.



Cluster A



Cluster B



Cluster C



Cluster D

1.
  - a. Which is the most compact cluster of dots?
  - b. Which is the least compact cluster of dots?
  - c. List these clusters in order from most compact to least compact.

Advanced High School Package 1



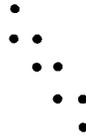
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Name \_\_\_\_\_

Date \_\_\_\_\_

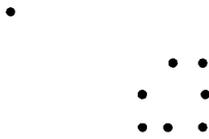
Here are four more clusters of eight dots each.



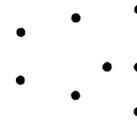
Cluster E



Cluster F



Cluster G



Cluster H

2.
  - a. Which is the most compact cluster of dots?
  - b. Which is the least compact cluster of dots?
  - c. List these clusters in order from most compact to least compact.
3. Describe a *procedure* or *algorithm* or *formula* for measuring the compactness of any cluster of dots. Use this procedure to assign a *numerical value* of compactness to the set of dots that you identified in 2a and 2b as the most and the least compact. Do your numbers seem reasonable in view of your original idea of compactness?



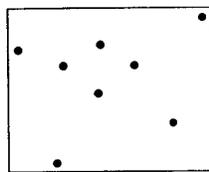
# A Sample Solution

7

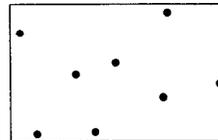
Students may answer questions 1 and 2 based upon intuitive notions of compactness, but in 3 they must formulate and apply a mathematical measure of compactness.

1. Cluster D appears the most compact, Cluster C appears the least compact, and the ordering of clusters is D, B, A, C.
2. Cluster F appears the most compact, Cluster G appears the least compact, and the ordering of clusters is F, E, H, G.
3. There are several possibilities for this measure, each having certain strengths and limitations. It is hard to say which of these definitions is the most desirable; ultimately this depends upon the purpose for which compactness is being defined.

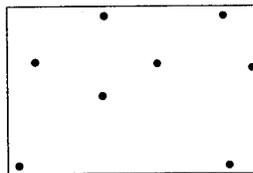
One solution is to define compactness in terms of the area of the smallest rectangle (with horizontal and vertical sides) that contains the dots.



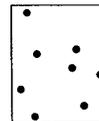
Cluster A



Cluster B

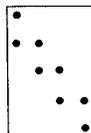


Cluster C



Cluster D

Here is what that solution looks like in the case of the second collection of dots.



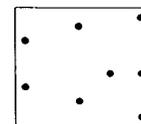
Cluster E



Cluster F



Cluster G

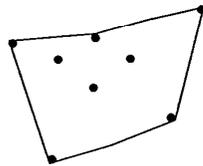


Cluster H

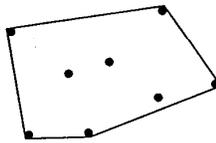
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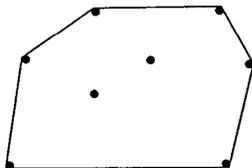
A second solution defines compactness in terms of area of the smallest (convex) polygon that can contain the set of dots. That solution looks like this:



Cluster A



Cluster B

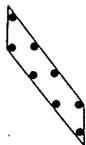


Cluster C



Cluster D

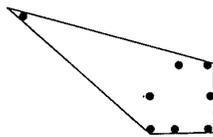
and



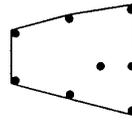
Cluster E



Cluster F



Cluster G



Cluster H

Other possible measures might be defined in terms of distance between points, for instance: the distance between the two most remote points, or the reciprocal of the sum of all the distances between pairs of points.

Finally, having chosen a measure of compactness, the student must show its application by computing numerical compactness values for the clusters selected in 2a and 2b, and assess the reasonability of these numbers. For example, students may find the distance between the two most remote points for each cluster. The distances could be measured as the following: E—2 cm, F—1.3 cm, G—3.2 cm, and H—2.1 cm. Students can then conclude that the most compact cluster is F and the least compact is G. A comparative remark about the two compact-ness numbers would be interesting but is not mandatory.

Compact-Ness

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