COMPLEX NUMBERS: A PREVIEW

Number Recording of the Egyptians and Greeks

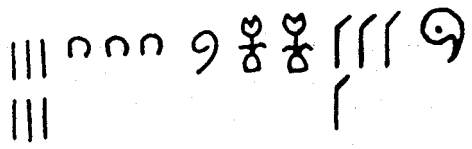
The spectacular emergence of the Egyptian government and administration under the pharaohs of the first two dynasties could not have taken place without a method of writing; and we find such a method both in the elaborate "sacred signs", or hieroglyphics, and in the rapid cursive hand of the accounting scribe. The hieroglyphic system of writing is a picture script, in which each character represents a concrete object, the significance of which may still be recognizable in many cases. In one of the tombs near the Pyramid of Gizeh there have been found hieroglyphic number symbols in which the number one is represented by a single vertical stroke, or a picture of a staff, and a kind of horseshoe, or heelbone sign ∩ is used as a collective symbol to replace ten separate strokes. In other words, the Egyptian system was a decimal one (from the Latin *decem*, "ten") which used counting by number systems is undoubtedly attributable to man's ten fingers and to his habit of counting on them. For the same reason, a symbol much like our numeral 1 was almost everywhere used to express the number one.

Special pictographs were used for each new power of 10 up to 10,000,000: 100 by a curved rope, 1000 by a lotus flower, 10,000 by an upright bent finger, 100,000 by a tadpole, 1,000,000 by a man holding up his hands as if in great astonishment, and 10,000,000 by symbol sometimes conjectured to be a rising sun.

1	10	100	1000	10,000	100,000	1,000,000	10,000,000

[Created by Nicole Leone, Newtown High School]

Other numbers could be expressed by using these symbols additively (that is, the number represented by a set of symbols is the sum of the numbers represented by the individual symbols), with each character repeated up to nine times. Usually, the direction of writing was from right to left, with the larger units listed first, then the others in order of importance. Thus, the scribe would write



to indicate our number 1 . 100,000 + .10,000 + 2 .1000 + 1 .100 + 3 .10 + 6 . 1 = 142,136.

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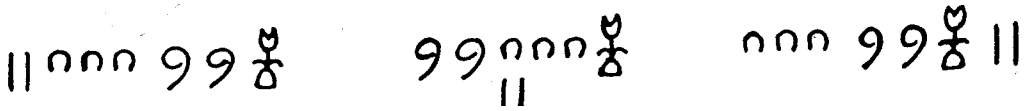
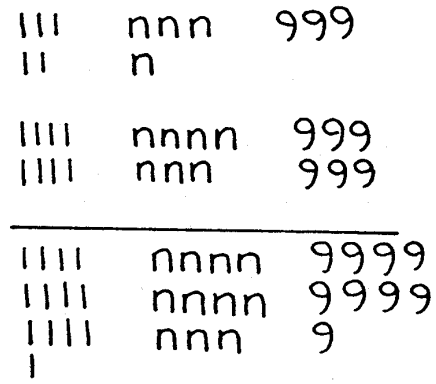
Occasionally, the larger units were written on the left, in which case the symbols were turned around to face the direction from which the writing began. Lateral space was saved by placing the symbols in two or three rows, one above the other. Since there was a different symbol for each power of ten, the value of the number represented was not affected by the order of the hieroglyphs within a grouping. For example,



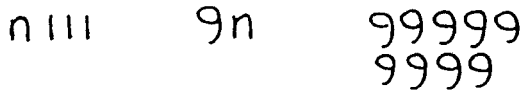
all stood for the number 1232. Thus the Egyptian method of writing numbers was not a positional system – a system in which one and the same symbol has a different significance depending on its position in the numerical representation.

Addition and subtraction caused little difficulty in the Egyptian number system. For addition, it was necessary only to collect symbols and exchange ten like symbols for the next higher symbol. This is how the Egyptians would have added, say, 345 and 678.

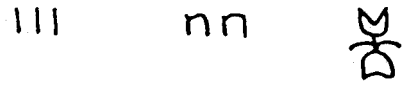
$$\begin{array}{r} 345 \\ \underline{678} \\ 1023 \end{array}$$



This converted would be

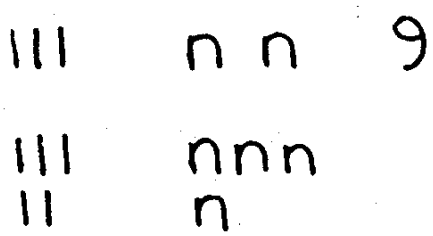


and converted again,

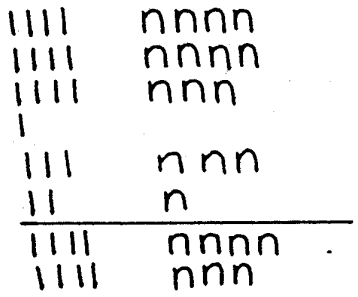


Subtraction was performed by the same process in reverse. Sometimes “borrowing” was used, wherein a symbol for the large number was exchanged for ten lower-order symbols to provide enough for the smaller number to be subtracted, as in the case

$$\begin{array}{r} 123 \\ -\ \underline{45} \\ 78 \end{array}$$



which, converted, would be



Although the Egyptians had symbols for numbers, they had no generally uniform notation for arithmetical operations. In the case of the famous Rhind Papyrus (dating about 1650 B.C.), the scribe did represent addition and subtraction by the hieroglyphs and , which resemble the legs of a person coming and going.

As long as writing was restricted to inscriptions carved on stone or metal, its scope was limited to short records deemed to be outstandingly important. What was needed was an easily available, inexpensive material to write on. The Egyptians solved this problem with the invention of papyrus. Papyrus was made by cutting thin lengthwise strips of the stem of the reedlike papyrus plant, which was abundant in the Nile Delta marshes. The sections were placed side by side on a board so as to form a sheet, and another layer was added at right angles to the first. When these were all soaked in water, pounded with a mallet, and allowed to dry in the sun, the natural gum of the plant glued the sections together. The writing surface was then scraped smooth with a shell until a finished sheet (usually 10 to 18 inches wide) resembled coarse brown paper; by pasting these sheets together along overlapping edges, the Egyptians could produce strips up to 100 feet long, which were rolled up when not in use. They wrote with a brushlike pen, and ink made of colored earth or charcoal that was mixed with gum or water. Thanks not so much to the durability of papyrus as to the exceedingly dry climate of Egypt, which prevented mold and mildew, a sizable body of scrolls has been preserved for us in a condition otherwise impossible.

The Rhind Papyrus starts with a bold premise. Its content has to do with "a thorough study of all things, insight into all that exists, knowledge of all obscure secrets." It soon becomes apparent that we are dealing with a practical handbook of mathematical exercises, and the only "secrets" are how to multiply and divide. Nonetheless, the 85 problems contained therein give us a pretty clear idea of the character of Egyptian mathematics. The Egyptian arithmetic was essentially "additive", meaning that its tendency was to reduce multiplication and division to repeated additions. Multiplication of two numbers was accomplished by successfully doubling one of the numbers and then adding the appropriate duplications to form the product. To find the product of 19 and 71, for instance, assume the multiplicand to be 71, doubling thus:

1	71
2	142
4	284
8	568
16	1136

Here we stop doubling, for a further step would give a multiplier of 71 that is larger than 19. Since 19 = 1 + 2 + 16, let us put checks alongside these multipliers to indicate that they should be added. The problem 19 times 71 would then look like this:

√	1	71
√	2	142
	4	284
	8	568
√	16	1136
totals		1349

Adding those numbers in the right-hand column opposite the checks, the Egyptian mathematician would get the required answer, 1349; that is,

$$1349 = 71 + 142 + 1136 = (1 + 2 + 16)71 = 19 \,.\, 71.$$

Had the number 19 been chosen as the multiplicand and 71 as the multiplier, the work would have been arranged as follows:

√	1	19
√	2	38
√	4	76
	8	152
	16	304
	32	608
√	64	1216
totals	71	1349

Since 71 = 1 + 2 + 4 + 64, one has merely to add these multiples of 19 to get, again, 1349.

The method of multiplying by doubling and summing is workable because every integer (positive) can be expressed as a sum of powers of 2; that is, as a sum of terms from the sequence, 1,2,4,8,16,32,….It is not likely that the ancient Egyptians actually proved this fact, but their confidence therein was probably established by numerous examples. The scheme given above is sometimes called Russian multiplication because of its use among the Russian peasants. The obvious advantage is that it makes memorizing tables unnecessary.

Egyptian division might be described as doing multiplication in reverse – where the divisor is repeatedly doubled to give the dividend. To divide 91 by 7, for example, a number x is sought such that 7x = 91. This is found by doubling 7 until a total of 91 is reached; the procedure is shown herewith.

	1	7
	2	14
	4	28
	8	56
totals	13	91

Finding that 7 + 28 + 56 = 91, one adds the powers of two corresponding to the checked numbers, namely, 1 + 4 + 8 = 13, which gives the desired quotient. The Egyptian division procedure has the pedagogical advantage of not appearing to be a new operation.

The numbers in the reading start at 1 and they are all numbers used for counting. These numbers make up the number system that we call Natural Numbers. Think about the way that the ancient Egyptians looked at these numbers and the way that the reading defined and described addition, subtraction, multiplication, and division of these numbers. Use this knowledge to try to fill in the following chart.

Operation	Ancient Egyptian Counting Numbers	Our Natural Numbers
Addition	***Description/Definition:*** ***Example:***	***Description/Definition:*** ***Example:***
Subtraction	***Description/Definition:*** ***Example:***	***Description/Definition:*** ***Example:***
Multiplication	***Description/Definition:*** ***Example:***	***Description/Definition:*** ***Example:***
Division	***Description/Definition:*** ***Example:***	***Description/Definition:*** ***Example:***

Answer the following questions:

1. When you add two Natural Numbers is the answer always another Natural Number? If so, why do you think that it is true? If not, give an example of two numbers that do not work.

2. When you subtract two Natural Numbers is the answer always another Natural Number? If so, why do you think that it is true? If not, give an example of two numbers that do not work.

3. When you multiply two Natural Numbers is the answer always another Natural Number? If so, why do you think that it is true? If not, give an example of two numbers that do not work.

4. When you divide two Natural Numbers is the answer always another Natural Number? If so, why do you think that it is true? If not, give an example of two numbers that do not work.

Eventually, Natural Numbers were not enough; more numbers were needed. Think of two situations; one purely mathematical and one practical, for which, to accurately describe the situations, you would need numbers that are not Natural Numbers.

Refine your definitions of all the operations on natural numbers to make sense for integers.

Operation	Natural Numbers	Integers
Addition	***Description/Definition:*** ***Example:***	***Description/Definition:*** ***Example:***
Subtraction	***Description/Definition:*** ***Example:***	***Description/Definition:*** ***Example:***
Multiplication	***Description/Definition:*** ***Example:***	***Description/Definition:*** ***Example:***
Division	***Description/Definition:*** ***Example:***	***Description/Definition:*** ***Example:***

Answer the following questions:

1. When you add two Integers is the answer always another Integer? If so, why do you think that it is true? If not, give an example of two numbers that do not work.

2. When you subtract two Integers is the answer always another Integer? If so, why do you think that it is true? If not, give an example of two numbers that do not work.

3. When you multiply two Integers is the answer always another Integer? If so, why do you think that it is true? If not, give an example of two numbers that do not work.

4. When you divide two Integers is the answer always another Integer? If so, why do you think that it is true? If not, give an example of two numbers that do not work.

Eventually, Integers were not enough; more numbers were needed. Think of two situations; one purely mathematical and one practical, for which, to accurately describe the situations, you would need numbers that are not Integers.

Redefine your definitions of all the operations on Integers to make sense for Rational Numbers.

Operation	Integers	Rational Numbers
Addition	***Description/Definition:*** ***Example:***	***Description/Definition:*** ***Example:***
Subtraction	***Description/Definition:*** ***Example:***	***Description/Definition:*** ***Example:***
Multiplication	***Description/Definition:*** ***Example:***	***Description/Definition:*** ***Example:***
Division	***Description/Definition:*** ***Example:***	***Description/Definition:*** ***Example:***

Answer the following questions:

1. When you add two Rational Numbers is the answer always another Rational Number? If so, why do you think that it is true? If not, give an example of two numbers that do not work.

2. When you subtract two Rational Numbers is the answer always another Rational Number? If so, why do you think that it is true? If not, give an example of two numbers that do not work.

3. When you multiply two Rational Numbers is the answer always another Rational Number? If so, why do you think that it is true? If not, give an example of two numbers that do not work.

4. When you divide two Rational Numbers is the answer always another Rational Number? If so, why do you think that it is true? If not, give an example of two numbers that do not work.