DESIGNING A NUMBER SYSTEM

Background: There is evidence that as early as 30,000 BC our ancient ancestors were tallying or counting things. That is where the concept of number systems began. The most basic number system is the one that can be used for counting. This system is called the Natural Numbers. The Natural Numbers begin at 1 and increase by 1 to create each new number. Quickly, it becomes clear that, though the Natural Numbers are the most practical, they are not the only numbers. If our system of numbers was limited to the Natural Numbers then a number such as –2 would have no meaning. The next number system is the Integers. Integers include the Natural Numbers, zero, and the negative Natural Numbers. However, this number system is not even enough, for if the only accepted number system was the Integers the number ½ would not exist. Therefore, the number system next grows to include all ratios of Integers. This system is called the Rational Numbers. This process continues as numbers such as π and $\sqrt{2}$, called Irrational Numbers, are added to the Rational Number system to create the Real Numbers. Today the largest accepted number system is the Complex Number system. The Complex Number system includes all real numbers as well as numbers such as $\sqrt{-3}$ which are undefined in the Real Number system. These are the only defined number systems to date, but does that mean they are the only number systems in existence?

Task: To complete this assignment you must take on the role of a group of mathematicians. You must learn all that you can about the existing number systems; study the origins of each system; study how the different mathematical operations are defined on each system. As you are researching these number systems look for patterns. As we begin to study each new system keep the following questions in mind. What was the reason for defining this number system? How does the origin of this number system compare to that of the others we have studied thus far? How are addition, subtraction, multiplication, and division defined on this system? How do these definitions compare to the corresponding definitions on the other number systems that we have studied? What are some of the characteristics of this number

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system and how do they compare to the characteristics that you have noticed in other number systems? What are the possible gaps in the system; i.e. what numbers or situations cannot be described using only the numbers in this system? After studying these systems, decide on a basis for a new number system and think about how you would define mathematical operations on that system. Once you have chosen a basis for your new number system you must thoroughly present your ideas in a formal research paper as well as in a short, persuasive presentation.

Audience: You must present your ideas to a panel, which will include your classmates, mathematicians from the community (if possible), and me. You must keep in mind that you are mathematicians and view this panel as a group of your peers in the mathematical community.

Purpose: Your purpose is to develop and defend your number system using what you have learned about other number systems.

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Procedures:

1. Each number has a basis. For instance, the basis for the Natural Number system is the number 1. It is the first number in the number system and every other number is generated by successively adding ones. The Rational Numbers are generated by the division of Integers. Complex Numbers are generated by $\sqrt{-1}$. Your first task is to choose a basis for your number system. You can do this in one of three ways. You can look at an existing number system for a different perspective; for instance, explain the Integers in a base 12 system rather than the familiar base 10 system. You could start with a completely fictitious number and build a number system around it. Finally, if you notice a gap in any existing number system, define a number that will fill this gap and extend the existing number system to include it.

2. After determining the basis for your number system, generate the rest of the numbers in the system.

3. Define addition, subtraction, multiplication, and division and any other relevant operations on this number system.

4. Write a paper describing the origins of your number system, clearly define the operations on this system, and defend the mathematical validity of your system, making comparisons to the number systems that we have studied. In addition, describe any unique characteristics of this system and any characteristics that it shares with other number systems.

5. Prepare a five to ten minute presentation defending your work.

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Performance Task Assessment List:

To receive a score of 88% on this project you must meet the following criteria. For each criterion you may receive up to 2 additional points if you were exceedingly insightful, thorough, creative, organized, prepared, etc. However, for each criterion that you do not meet you may lose up to 6 points.

Criterion	Teacher Assessment	Self Assessment
1. You explained the origin of your number system and compared it to the origin of the other number systems that you have studied.		
2. You included a clear and thorough description of the numbers in your number system and described how they are formed.		
3. You either described how your number system fits into the existing number systems or you explained how it works as a stand-alone number system.		
4. You defined addition, subtraction, multiplication, division, and any other relevant operations on this number system and compared those definitions to those you learned about for the other number systems.		
5. You provided evidence, practical or purely mathematical, that supports the claim that this number system will enrich the study of mathematics.		
6. You worked together, as a group, to construct a clear and cohesive presentation of your findings, a presentation that is both concise and persuasive.		

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