Task 4

Designing a Staircase

Long Task

Overview

Use the concept of slope in an applied context involving staircases.

Operate with an inequality that limits the overall size of an allowable stair step.

Figure out a choice of rise and run that meets given requirements on stair size and slope.

Task Description

The task presents guidelines on step size and steepness allowable in staircases. The students are then asked to design a staircase that joins one floor with another floor 11 feet above it. The main job is to determine how many steps are required, and what size they must be.

Assumed Mathematical Background

Students should have done some work with the concept of slope of a line thought of as the (vertical) rise divided by the (horizontal) run of any section of the line.

Core Elements of Performance

- use the concept of slope (rise over run) in the setting of a staircase of repeated equal steps
- work with inequalities that specify the minimum and maximum allowable slope
- work with inequalities that specify minimum and maximum step size (where step size is measured as twice the rise plus the run)
- find the dimensions of a step that is within the guidelines and that can be used to span a given vertical distance

Circumstances

Grouping:	Students complete an individual written response.
Materials:	ruler and calculator
Estimated time:	45 minutes

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Name Date

Designing a Staircase

This problem gives you the chance to

- ***design a staircase that meets certain guidelines***
- ***use the concept of slope in a practical situation***

Design a staircase that has a total rise of 11 feet and that meets the design guidelines given below.

Communicate your design decisions clearly: how many risers and treads are there, and what size are they?

Include your calculations.

Show how each design guideline is met.

Design Guidelines

- The slope of the staircase must be between 0.55 and 0.85.
- Twice the rise plus the run must be between 24 and 25 inches.
- There can be no irregular steps: Each step must be the same size.

Some useful terms

tread: the horizontal part of a step

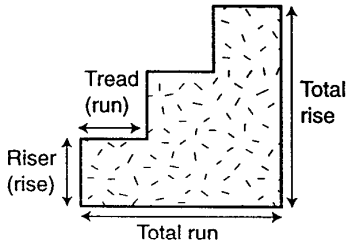
run: the length of the tread

riser: the vertical part of a step

rise: the height of the riser

slope: a measure of the steepness of a staircase found by dividing the riser height (rise) by the tread length (run):

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$



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A Sample Solution

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There are many approaches to this problem. One is to guess a number of steps, say 20 steps. Since the stairs must rise 11 feet = 132 inches, each riser is 132 ÷ 20 = 6.6 inches in height. It is required that twice the riser (13.2 inches) plus the tread must be between 24 and 25 inches. Let's choose 24.5 inches. This makes the tread 11.3 inches. What slope is this? It is 6.6 ÷ 11.3 ≈ 0.58. This is within the required limits 0.55 ≤ slope ≤ 0.85. So this design works.

Summary: There are 20 steps, each with the dimensions riser = 6.6 inches and tread = 11.3 inches. The slope is about 0.58.

Another Sample Solution

Here is a more systematic solution with more of the reasoning supplied:

Use the notation m = slope, R = riser height (in inches), and T = tread length (in inches).

Since slope = $m = \frac{\text{rise}}{\text{run}} = \frac{\text{riser height}}{\text{tread length}} = \frac{R}{T}$, the relation $R = mT$ always holds.

Choose an arbitrary slope m for the staircase within the given range, say $m = 0.75$. Then $R = 0.75T$.

The requirement "twice the rise plus the run must be between 24 and 25 inches" can be expressed as: 24 inches ≤ $2R + T \leq 25$ inches. Substituting for R and solving for T, we get:

$24 \leq 2(0.75T) + T \leq 25$
$24 \leq 1.5T + T \leq 25$
$24 \leq 2.5T \leq 25$
$9.6 \leq T \leq 10$

Since $R = 0.75T$, 7.2 inches ≤ $R \leq 7.5$ inches.

The steps altogether must rise 11 feet (or 132 inches), and the individual steps must all be the same size. This means 132 ÷ R = a whole number. Checking the values of R at the two extremes:

132 inches ÷ 7.2 inches = 18.3 steps and 132 inches ÷ 7.5 inches = 17.6 steps.

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That means our staircase must have 18 steps. Since all the steps must be the same height, $R = 132 \div 18 = 7\frac{1}{3}$ inches, and $T = \frac{R}{0.75} = 9\frac{7}{9}$ inches.

Summary: There are 18 steps, each with the dimensions $R = 7\frac{1}{3}$ inches, and $T = 9\frac{7}{9}$ inches. The slope is 0.75.

We can double-check that our staircase meets the guideline 24 inches $\leq 2R + T \leq 25$ inches: $2\left(7\frac{1}{3}\right) + 9\frac{7}{9} = 24\frac{4}{9}$ inches.

Since 24 inches $\leq 24\frac{4}{9}$ inches ≤ 25 inches, the staircase is within the guidelines.

Still Another Sample Solution

A more thorough treatment comes through starting with a number N of steps, finding the required riser height $R = 132 \div N$, finding the maximum and minimum tread length T for this value of R using the requirement 24 inches $\leq 2R + T \leq 25$ inches, and computing the slope for each of these two tread lengths. This can be done for all the values of N that lead to acceptable slopes, and the result can be put in a table.

The key is in the computed slope. Those slopes that fall within the limits of 0.55 and 0.85 are entered in bold. These represent possible staircases. (Values are rounded off.)

N	$R = \frac{132}{N}$	$T = 24 - 2R$	slope $= \frac{R}{T}$	$T = 25 - 2R$	slope $= \frac{R}{T}$
16	8.25	7.5	1.1	8.5	0.97
17	7.76	8.48	0.92	9.48	**0.82**
18	7.33	9.34	**0.78**	10.34	**0.71**
19	6.95	10.1	**0.69**	11.1	**0.63**
20	6.6	10.8	**0.61**	11.8	**0.56**
21	6.29	11.42	**0.55**	12.42	0.51

The two extremes of tread length can be specified further:

The shortest tread comes for $N = 17$ steps, $R = 7.76$ inches. We can just set $\frac{R}{T} = 0.85$, the maximum allowed slope, and get $T \approx 9.13$ inches. As a check, $2R + T = 15.52 + 9.13 \approx 24.65$, which is less than the maximum of 25.

The longest tread comes for $N = 21$ steps, $R = 6.29$ inches. Here we can set $\frac{R}{T} = 0.55$, the minimum allowed slope, and get $T \approx 11.4$ inches.