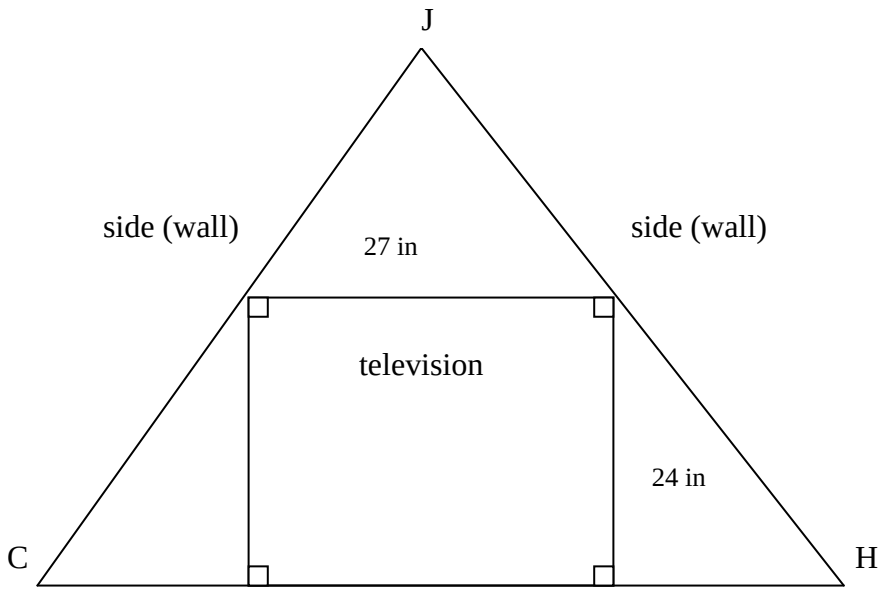
ENTERTAINMENT CENTER

In designing a new corner cabinet for our family room, my family and I had to figure out how deep to make it so that the TV we currently have would fit. We want the new cabinet to be the same length on each side (along the two walls). Here is an overhead view:



How long should each side of the cabinet be? Show all of your calculations and explain how you know you are correct.

[Source: http://www.exemplars.com/math_sample_9-12.html]

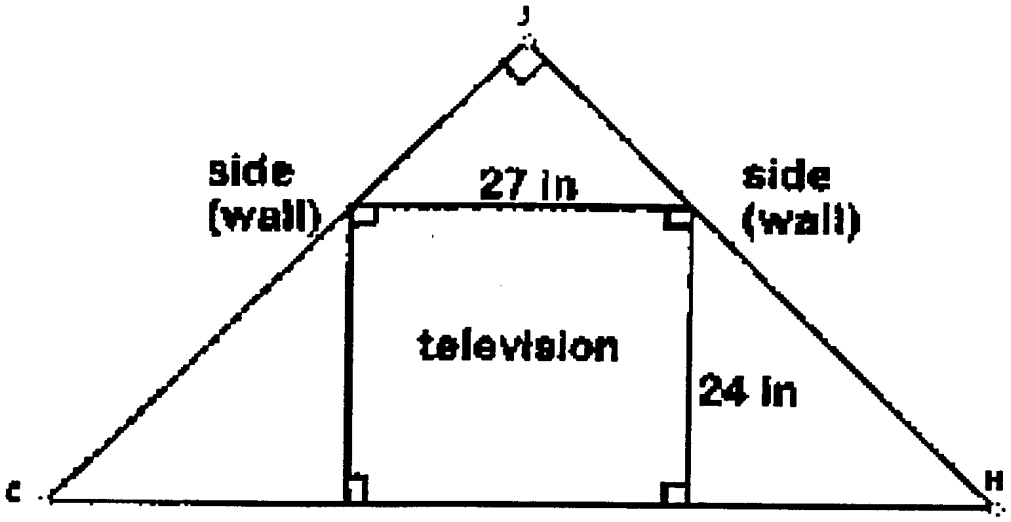
Entertainment Center

In designing a new corner cabinet for our family room, my family and I had to figure out how deep to make it so that the TV we currently have would fit. We want the new cabinet to be the same length on each side (along the two walls).

Here is an overhead view:

How long should each side of the cabinet be?

Show all of your calculations and explain how you approached and solved this problem.



Context

In exploring similar and special right triangles, a variety of approaches were used. The unit began with a discussion of different figures on the overhead that contained different types ofsimilarity and lack thereof, allowing us to define what it is that we mean by indicating that figures are similar.

Once a common understanding was reached, we went to triangles and discovered:

a) a segment parallel to one side of a triangle breaks it into proportional lengths and similar triangles;

b) a segment which connects the midpoints of two sides of a triangle is parallel to the third side and has a measurement equal to one half the length of the third side.

[Source: http://www.exemplars.com/math_sample_9-12.html]

A journal entry was done with the following diagram included:

"Given the drawn triangle relationships, show (give a convincing argument) that the three triangles are similar or not."
We explored the relationships in this diagram using other problems similar to the enclosed task in groups, with the enclosed task being the one used for assessment.

The Pythagorean Theorem was already familiar to the students, so we used it to discover the special right triangle relationships for the 45-45-90 and 30-60-90 triangles.
Before assigning this task, we did have a few days of introduction into trigonometry. This consisted of looking at similar triangles as a class and recognizing that the corresponding ratios of sides always stayed the same. They concluded that these ratios could be related to one of the angles that was the same as another in a similar right triangle. Scientific calculators were introduced, as well as trigonometric tables. Students enjoy working with technology, and it helped us to get through the drudgery of reading trigonometric tables.

What This Task Accomplishes
This tasks puts the student in the role of designer, using specifications from a diagram. They must employ (a variety of) techniques and develop appropriate strategies for solving the problem. A meta-cognitive aspect is built into the task by requiring an explanation of the approach and consequent solution.

What The Student Will Do
The students work individually on this problem during a class period. They must show all their work and then verbalize their work and the results. The student is free to choose a variety of approaches and tools.

Time Required For The Task
The students had thirty minutes to complete the task, and it seemed sufficient.

Interdisciplinary Links
As with most of my performance tasks, this one comes from my real life experience. I believe my students enjoy that connection and work hard on the problems I have had to wrestle with myself.

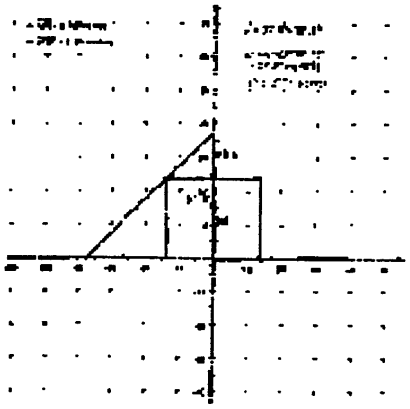
Teaching Tips
The students enjoyed the task for the most part, though some of my accelerated students had difficulty getting started because they could not see an easy way to an answer. Some students finished early, and I encouraged them to keep writing and to look for alternative approaches. To my surprise, many students asked for scientific calculators to work on this task, even though I had originally intended for it to be a similar and special right triangle problem. Not much time had been spent on trigonometry prior to the task, so it pleased me to find some wanting to give it a try.

Concepts To Be Assessed and Skills To Be Developed
Pythagorean Theorem

[Source: http://www.exemplars.com/math_sample_9-12.html]

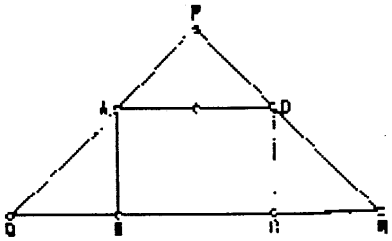
Ratios in "special" right triangles (45-90-45)
Generating and solving proportions
Using right triangle trigonometry
Using trigonometric functions on the calculator
Problem-Solving
Communication/Writing Mathematics
Suggested Materials
Calculators, graph paper, trig tables, rulers, protractors, computer with software such as the Geometer's SketchPad 3.0.

Possible Solutions



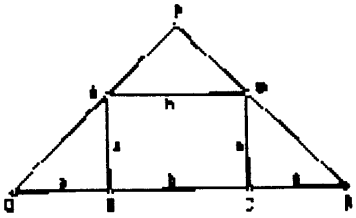
1. Using Geometer's SketchPad 3.0, one can set up this problem in a coordinate system. If a proper scale is used, the distances can simply be measured. This kind of solution would show that the student understands the symmetry of this problem and makes handy use of this. Even though many students identify the cabinet to be an isosceles triangle, most of them focus on the smaller interior triangles and not on the symmetry in the whole. A coordinate approach might stimulate students to use the symmetry of the problem.

2. RD = 2.06 Inches QP2. 2 = 25.604 inches
DP = 1.5 Inches QR 2 = 25.604 inches
QP = 3.58 Inches
QR = 5.06 Inches



By drawing any situation like the one proposed in the drawing, one can use the Geometer's SketchPad to discover that the Pythagorean theorem is useful here, as illustrated on the left. Of course, students can do an accurate drawing by hand and verify the theorem.

This can be generalized in the following situation:



Since $PD = \frac{1}{2}b\sqrt{2}$ and $DR = a\sqrt{2}$ it follows that:

$PR = \frac{1}{2}b\sqrt{2} + a\sqrt{2} = \left(\frac{1}{2}b + a\right)\sqrt{2}$

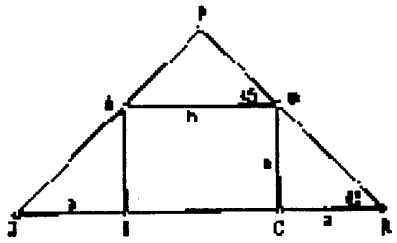
If you observe these results in triangle PQR it follows that:

$PR^2 * 2 - QR^2 \rightarrow PR^2 = \frac{1}{2}(b+2a)^2 \Rightarrow$

$PR = \sqrt{\frac{1}{2}} * (b+2a) \Rightarrow PR = \sqrt{2} * \left(\frac{1}{2}b + a\right)$

[Source: http://www.exemplars.com/math_sample_9-12.html]

3. Next follows a result using trigonometric ratios:



sin (45°) = 0.707
cos (45°) = 0.707
0.5 · (2)0.5 = 0.707

Point of interest here is that triangles APD and DCR have different parts given, the hypotenuse and a leg respectively. Since the value for the sine and cosine of 45 degrees are the same a student could end up with a correct answer using an incorrect choice of function. This makes the written work of the student more valuable.

These are possible solutions:

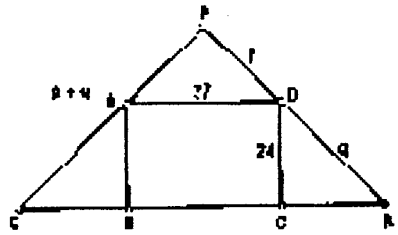
$$\frac{PD}{b} = \cos(45) \Rightarrow PD = b \cdot \frac{1}{2}\sqrt{2} \quad (\Delta APD)$$

$$\frac{a}{DR} = \cos(45) \Rightarrow DR = a \cdot \sqrt{2} \quad (\Delta PDC)$$

$$\Rightarrow PR = \sqrt{2} \cdot (\tfrac{1}{2}b + a)$$

4.

Using proportions in this case will be algebraically cumbersome, but might be an appropriate challenge for some advanced students.



Students quickly find that QR=75. We will develop two proportions to solve for p and q so we can find QP=RP=p+q.

In comparing triangles PAD and PQR we come to the following proportion

$$\frac{27}{75} = \frac{p}{p-q}$$

In comparing triangles PAD and CDR we come to the following proportion

$$\frac{24}{p} = \frac{q}{27}$$

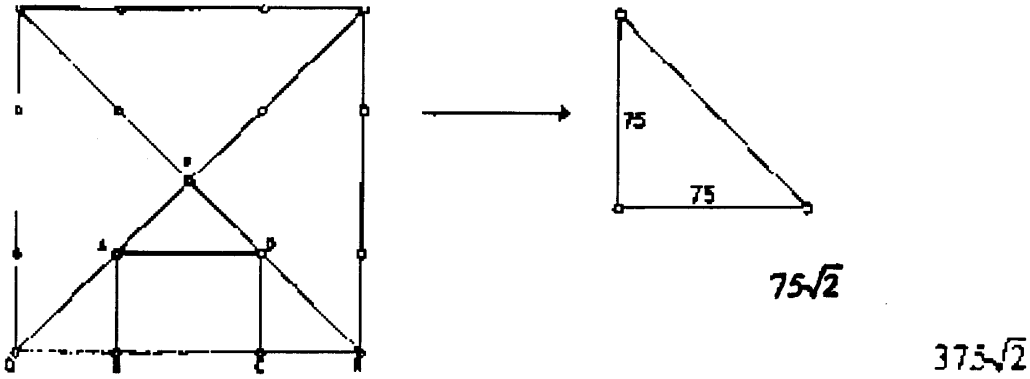
In comparing triangles PQR and CDR we come to the following proportion

$$\frac{24}{p+q} = \frac{q}{75}$$

[Source: http://www.exemplars.com/math_sample_9-12.html]

5.

The next solution is very quick and elegant. It requires the student to extend the figure in question beyond itself. If one constructs a square with four of these cabinets then each side of this square would be 75 inches. The sides of the cabinet would then be half the measure of the diagonal, as illustrated in the figure below.



Extensions

Some extensions are possible to this problem. You can ask students to take the thickness of the wood into consideration when answering this problem. Also what about beveling the cuts? And how could an entire cabinet be made from 4 feet by 8 feet sheets of plywood with the least amount of waste?

Task Specific Rubric/Benchmark Descriptors

Novice	Use of correct formulas is minimal or non-existent. Written explanations express confusion and incorrect information. Diagram work is sparse and incorrect. Sometimes no final answer is given.
Apprentice	Has some accurate formulas to find partial correct solutions to the problem. Written explanations usually include a point at which the student becomes stuck. Diagram work includes some incorrect information at times.
Practitioner	Uses accurate formulas to find correct solutions to the problem. His/her written explanations are clear and straightforward. Diagram work is accurate.
Expert	Uses multiple approaches to the task to find correct solutions. Written explanations show the student's thoughts clearly. Diagram usage is solid and correct.

Author

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Level	Understanding	Strategies, Reasoning, Procedures	Communication
Novice	• There is no solution, or the solution has no relationship to the task. • Inappropriate concepts are applied and/or procedures are used.	• No evidence of a strategy or procedure, or uses a strategy that does not help solve the problem. • No evidence of mathematical reasoning. • There were so many errors in mathematical procedures that the problem could not be solved.	• There is no explanation of the solution, the explanation con not be understood or it is unrelated to the problem. • There is no use or inappropriate use of mathematical representations (e.g. figures diagrams, graphs, tables, etc.) • There is no use, or mostly inappropriate use, of mathematical terminology and notation.
Apprentice	• The solution is not complete indicating that parts of the problem are not understood.	• Uses a strategy that is partially useful, leading some way toward a solution, but not to a full solution of the problem. • Some evidence of mathematical reasoning. • Could not completely carry out mathematical procedures.	• There is an incomplete explanation; it may not be clearly presented. • There is some use of appropriate mathematical representation. • There is some use of mathematical terminology and notation appropriate of the problem.
Practitioner	• The solution shows that the student has a broad understanding of the problem and the major concepts necessary for its solution.	• Uses a strategy that leads to a solution of the problem. • Uses effective mathematical reasoning. • Mathematical procedures used	• There is a clear explanation. • There is appropriate use of accurate mathematical representation. • There is effective use of mathematical terminology and notation.
Expert	• The solution shows a deep understanding of the problem including the ability to identify the appropriate mathematical concepts and the information necessary for its solution.	• Uses a very efficient and sophisticated strategy leading directly to a solution. • Employs refined and complex reasoning • Applies procedures accurately to correctly solve the problem and verify the results	• There is a clear, effective explanation detailing how the problem is solved. All of the steps are included so that the reader does not need to infer how and why decisions were made. • Mathematical representation is actively used as a means of communicating ideas related to the solution of the problem. • There is precise and appropriate use of mathematical terminology and notation.

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