Name   Date

MasterMind

This problem gives you the chance to

- ***count the number of possible combinations in a game***
- ***modify the solution for games with slightly different rules***
- ***explain how to generalize the solution***

The object of the game **MasterMind** is to guess the colored pegs that make up your opponent's secret code. There are 6 colors of pegs (white, yellow, red, green, blue, and black), and the code consists of 4 pegs. One possible code would be:

(red) (blue) (green) (red)

The object of the game is to guess the code exactly, in the correct order. You are allowed ten guesses, and after each guess you receive some information from the code setter about how close you are getting.

1. How many possible secret codes are there in **MasterMind**?

2. Some people play **MasterMind** with the rule that all the pegs in the secret code must be of different colors. This would rule out the example above. How much does this change the game? (Compare the number of possible codes under this rule to the number in the original game.)

3. Other people make the game of **MasterMind** harder by allowing blank spaces in the code, effectively creating a "7th color." For instance, this code would be allowed:

   (yellow) (blank) (black) (white)

   How does this change the game?

4. The instruction booklet for **MasterMind** actually contains a chart showing how many possible codes there are, for any number of colors from 1 to 7, and allowing or disallowing repeated colors. Explain how you would go about making such a table. (You don't have to calculate all of the values.) What will be the smallest and largest values in the table?

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A Sample Solution

Task 6

1. There are 6 color choices for every position, so there are $6 \times 6 \times 6 \times 6 = 1296$ possibilities.

2. Here the possible combinations may be counted as follows: any of the 6 colors may be chosen for the first peg. Having made this choice, any of the 5 remaining colors may be chosen for the second place. Continuing, there are 4 choices for the next place and then 3 choices for the final peg. The total number of possibilities is $6 \times 5 \times 4 \times 3 = 360$, so the number of secret codes is about a fourth of the number in the first game.

3. There are two good answers to this question, depending upon whether one considers the game with repeated colors (as in 1) or without repeated colors (as in 2). The same reasoning from 1 and/or 2 continues to apply here.

   Using 7 colors with repeats allowed, there are $7 \times 7 \times 7 \times 7 = 2401$ codes.

   With the addition of a 7th color, but no repeated colors, there are $7 \times 6 \times 5 \times 4 = 840$ codes.

   In either case, there are about twice as many combinations with 7 colors as with 6.

4. In the game without repetition, there must be at least 4 colors, or it is impossible to make any valid codes. If there are $n \geq 4$ colors in the game, there are $n(n-1)(n-2)(n-3)$ combinations to be made without repeats.

   In the game with repetitions, there can be any number n of colors, and there will be n^4 possible codes.

   The smallest value in the table is either 0 (if one includes the games with less than 4 colors and no repetitions, for which there are no possible codes), or 1 (for 1-color game with repetitions). These are not particularly interesting games.

   The game with the most combinations allows all 7 colors with repetitions, yielding $7^4 = 2401$ combinations.

   Students are not required to complete a table of values, but it is shown on the next page.

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Task 6

Number of colors	Combinations without repetitions	Combinations with repetitions
n	$n(n-1)(n-2)(n-3)$, when $n \geq 4$	n^4
1	0	1
2	0	16
3	0	81
4	24	256
5	120	625
6	360	1296
7	840	2401

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