

Developing Estimation Skills in the Primary Grades

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A group of primary children was asked to estimate the number of seeds in a pumpkin. After cutting the top off so the children could see the seeds inside, the teacher carried the pumpkin around the room. Each child looked in and made a guess. The guesses of these six-year-olds ranged from one to one million because of their limited experience with number.

When young children try to estimate, predict, or mentally compute, they often suggest or accept answers that adults would consider unreasonable. Their limited perception of the numbers involved and their limited number sense affect their guesses. Young children often think that a number such as 35 is very large and might see little difference between 35 and 1000. To them, 35 and 1000 are both very big numbers.

As early as kindergarten, many opportunities exist or can be created to help children develop an experiential base from which better estimates can be made. As the children are learning estimation skills they are also developing a better sense of number. Teachers can facilitate this development by having children estimate an unknown quantity by (1) comparing it to a known quantity, (2) partitioning it into known quantities, and (3) using mental computation.

A TEACHING STRATEGY FOR ESTIMATION

You can begin estimation activities by having the children make guesses about the quantity of a group of objects or the measure of an object. Questions involving how many, how much, how far, how long, and what fraction can be a starting point. After the children have made their initial estimates, you can give them a hint and let them revise their estimates on this basis.

One type of hint involves comparing the unknown to a known quantity; that is, showing an object or groups of objects similar in size or quantity and stating its numerical value. The children can then compare this known quantity to the unknown one. For example, you can hold up a book (with, say, 156 pages) and ask, "How many pages do you think there are in this book?" After the children have guessed, hold up another book for comparison. "There are 138 pages in this second book. Do you think there are more or fewer pages in the first book? Do you think there are more or fewer than 138 pages in the first book? Do you think there are just a few more pages in the first book or are there lots more? Does the first book have twice as many pages? Do you think the first book has 500 pages?...300 pages?...200 pages?" Through questions or hints you can direct the children's thinking, leading them to make better estimates.

A second type of hint is to partition the unknown into known quantities; that is, show a small part of the unknown and state its numerical value. The children can then estimate the number of such parts in the whole and count or mentally compute to derive a better estimate. For

example, hold up a book with 412 pages and ask, “How many pages do you think are in this book?” After the children have guessed, open the book to page 100. “Here are the first 100 pages. Do you think there are just a few more pages or a lot more? There are 100 pages from the beginning of the book to this page. Do you think there are more or less than 100 pages from this page to the end of the book? How many groups of 100 pages are there? Do you think there are 200 pages?...300?...400?...500?”

An activity should not end just because the children have arrived at a reasonable estimate. Strengthen estimation skills by having children verbalize the thinking they used to derive their answers. It helps make them more aware of the procedures they used, and it exposes thinking patterns to other children who may not be using them yet. After the children have made and refined their estimates, you can probe into the thinking behind the guesses. For example, “What were you thinking when you wrote that answer?” “When I showed you the second book with 138 pages, how did you decide how many more pages were in the first book?” “When I showed you 100 pages, how did you use that to help you decide how many pages were in the book?” “Did anyone think about it in a different way?” At first the children will have difficulty putting their thoughts in words, but continued questioning will result in clearer and more frequent responses.

Ask children often to write their estimates. The participation of each child is important. Having them write their answers makes everyone accountable. In addition, after you have given them a hint, they can refer to the written estimate and refine it on the basis of the new information.

Avoid treating estimates as right or wrong, but recognize that some estimates are better than others. Ask questions that indicate boundaries for the range of reasonable estimates. For example, if the known quantity is obviously smaller than the unknown, then estimates should be greater than the number given in the hint. For each estimation activity a range of reasonable guesses should be given. The children should be told why that range was chosen. The size of this range depends on the size of the object being estimated and the hint given. As the children’s estimation skills improve, the range for acceptable guesses should become narrower.

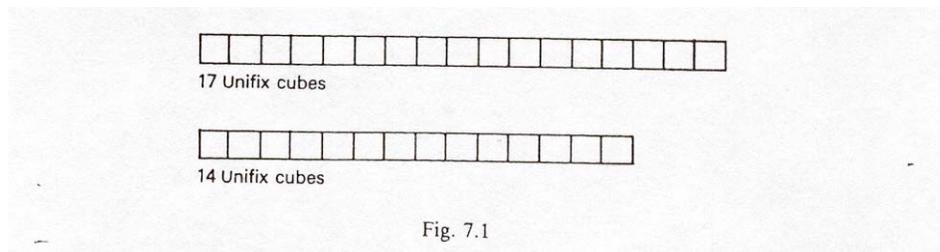
The size of the numbers used should be determined by the familiarity children have with those numbers. Typically, numbers to 30 can be used in kindergarten, numbers to 100 in grade 1, numbers to 1000 in grade 2, and numbers greater than 1000 in grades 3 and 4. In the activities that follow, change the numbers to match the abilities of the children.

ACTIVITIES FOR ESTIMATING AND COMPARING TO A KNOWN QUANTITY

Example 1

- | Do | Say |
|--|--|
| 1. Show 17 Unifix cubes linked together. Do not let the students count them. | “How many cubes do you think there are? Write your guess.” |
| 2. Now show 14 Unifix cubes linked together. Hold them or place them | “Here are 14 cubes. Now how many do you think there are in the first |

near the first group for no more than two seconds so the children will not have time to count to find the difference (fig. 7.1). group?"



3. Now remove the 14 Unifix cubes from sight. Leave the 17 cubes in view.
 - “How many cubes were in the second group?” (14)
 - “Are there more or fewer in this group?” (More)
 - “Are there more or less than 14?” (More)
 - “Are there just a few more, or a lot more?” (A few more)
 - “How many do you think are in this group?...Do you want to change your guess?...Write down your new guess.”

4. Now show the 14 Unifix cubes beside the 17 cubes again.
 - “Kevin, you wrote 16. How did you decide?”
 - “Who arrived at their estimate in a different way?”
 - “What did you think?”

An acceptable range of guesses would be 16 to 18.

Example 2

1. Sort out the dark brown and the green M & M’s from a package and place them in a transparent container. Show them to the children (fig. 7.2).
 - “What do I have in the container?” (Dark brown and green M & M’s)
 - “How many dark brown M & M’s do you think there are in the container? Write the number of your estimate.”

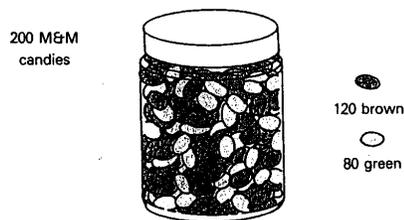


Fig. 7.2

2. Tell how many M & M's there are in the container.

“There are about 200 M & M's in the container. Now how many dark brown M & M's do you think there are? Write down a new estimate if you like.

3. Ask questions to help the children refine their answers.

“Are there more or fewer than 200 dark brown M & M's (Fewer)
 “Is the number of dark brown M & M's close to 200?” (Yes, it's close.)
 “Are more than half of the M & M's dark brown?” (Yes)
 “Are more than 100 M & M's dark brown?” (Yes)
 “Audra, you wrote down 168. How did you think to get that number? Who thought about it in a different way? Would it help to know how many green M & M's there are?” (Yes)
 “If there are 50 green M & M's, how many dark brown M & M's do you think there are?... That's right, Yolanda, 200 minus 50 is 150, so 150 would be a good estimate.”

An acceptable range of guesses would be 120 to 180.

Here are some other estimation activities that involve comparing an unknown to a known quantity.

1. Show the children two stacks of cards. Tell them how many are in one of the stacks. Ask them to estimate the number in the other stack.

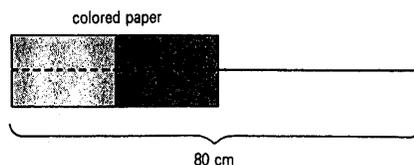
2. Have two children of different heights come to the front of the class. Measure the height of one of them. Have the rest of the children estimate the height of the other. Repeat this activity, but use weight instead of height.

3. Show the children two sets of coins. Tell the value of one set. Have them estimate the value of the other.
4. Briefly show a set of coins consisting of quarters, dimes, nickels, and pennies. Tell how many quarters there are. Have the children estimate the value of the set of coins.
5. Have the children close their eyes and raise their hands when they think one minute has elapsed. Repeat, only this time tell the children when a half minute has elapsed.
6. Ask the children to guess the following fractions: What part of the class is left-handed? What part of the class is wearing red? What part of the class is wearing glasses? Is each quantity closer to one-fourth, one-half, or three-fourths?

ACTIVITIES FOR ESTIMATING BY PARTITIONING INTO KNOWN QUANTITIES

Example 1

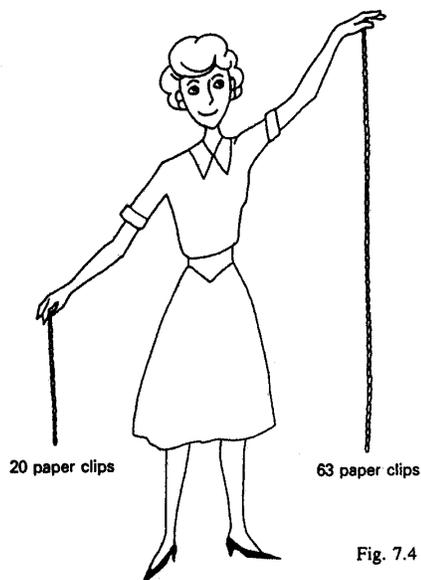
- | Do | Say |
|---|---|
| 1. Draw a line segment of about 80 cm on the chalkboard. Cut several strips of paper from the bottom of a standard sheet of paper. Tape one strip at one end of the line segment. | “How many strips of paper this length could we tape along the line segment? Write your guess.” |
| 2. Help the children find the middle of the line segment. Mark it on the chalkboard. | “Where is the middle of the line segment? How many strips of paper do you think it would take to reach to the middle?” (2) |
| 3. Tape another strip of paper to the board (fig. 7.3). | “Did two strips of paper reach about to the middle of the line segment?” (Yes)
“If two strips of paper reach to the middle, how many strips will it take to reach to the end?” (4) |



- | | |
|--|---|
| 4. Tape two more strips of paper along the line to show that four strips is about as long as the line segment. | “How many strips of paper did it take to reach to the end of the line segment?” (4) |
|--|---|

Example 2

1. Make a chain of about 63 paper clips. Show the children.
2. Show them a chain of 20 paper clips, holding it beside the longer chain (fig. 7.4).
3. Have children imagine putting other chains of 20 paper clips beside the long chain.



“How many paper clips do you think there are? Write down your estimate.”

“There are 20 paper clips in this chain. Now how many do you think there are in the long chain? Do you want to change your guess?”

“Imagine putting other chains of 20 paper clips beside the long chain. How many would it take to be just about as long?” (3)

“How many paper clips would 3 chains of 20 be?” (60)

“How did you figure that out?”

(Added $20 + 20 + 20$)

“Who did it in a different way?... How did you do it?”

Example 3

1. Show the children a jar of 400 or 500 pennies.
2. Remove 50 pennies from the jar (fig. 7.5).
3. Remove another 50 pennies from the jar.

“How many pennies do you think are in this jar? Write down your estimate.”

“I just took 50 pennies from the jar. Now how many pennies do you think there are in all? Remember to think of all the pennies.

“I just took another 50 pennies from the jar. Now how many pennies do you think there are in all? How many pennies have I taken out?” (100)

“Could I take out 100 more pennies?” (Yes)

“How many groups of 100 do you think I could take out? About how many pennies do you think there are in all? Explain your thinking.

Here are some more estimation activities that involve partitioning unknown into known quantities.

1. Show the children a stack of about 100 sheets of paper. Count out 20 sheets from the stack. Ask them to estimate the number of sheets of paper in all.
2. Ask the children to estimate the length of the teacher’s arm in centimeters. Now place a 10-cm rod beside your outstretched arm and have them estimate again.
3. Give the children four note cards to place on their desk edge to edge. Ask them to estimate how many note cards it would take to cover their desk.
4. Show about 30 nickels. Move five aside and count them. Ask how many nickels there are altogether. Ask how much money that is.

ACTIVITIES FOR ESTIMATING BY USING MENTAL COMPUTATION

Mental computation is not a separate estimation process; rather, it is sometimes needed when children estimate by comparing and partitioning with known quantities.

Children need instruction and practice with mental computation before they can use it efficiently for estimation. Following are some activities to show how mental computation can be developed using concrete materials. The emphasis is on verbalizing the step-by-step thinking used while the materials are being manipulated. These activities have to be repeated many times before the children are able to use the mental computations spontaneously. With two or three examples twice a week for a month, children should improve remarkably in their mental computation skill. These examples were selected to show instructional procedures for teaching mental computation. Children need to possess prerequisite skills, such as basic addition and multiplication facts and coin recognition, before attempting the mental computations described.

Adding or Subtracting Multiples of 10

Many estimations are simplified if children can mentally add or subtract a multiple of ten.

- | Do | Say |
|--|---|
| 1. Use bundling sticks to show 34. | “How many tens are there?” (Three)
“How much is 3 tens?” (Thirty)
“How many ones are there?” (Four)
“What number is shown?”
(Thirty-four) |
| 2. Show two more bundles of ten (fig. 7.6) | “How many tens are there in this pile?” (Two)
“What number is 2 tens?” (Twenty) |

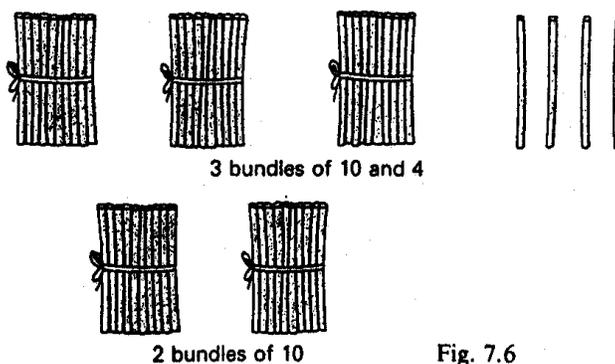


Fig. 7.6

- | | |
|---|---|
| 3. Move the 2 tens beside the 3 tens. Point to the 3 tens and the 2 tens when you ask how much is 30 plus 20. | “How much is 3 tens and 2 tens?” (5 tens)
“How much is 30 plus 20?” (Fifty)
“What is 50 plus 4?” (Fifty-four)
“To add 34 and 20, think 3 tens plus 2 tens is 5 tens, or fifty. Then add 4 more to get fifty-four.” |
|---|---|

Adding Single-Digit Numbers

Many mental computations involve adding a single-digit number to a two-digit number. Children usually count on by ones to get the answer; however, they can learn to quickly add the numbers mentally.

- | Do | Say |
|--|---|
| 1. Use 3 dimes and 8 pennies to show 38 cents. | “How many dimes are there?” (Three)
“How much is 3 dimes?” (Thirty cents)
“How many pennies are there? (Eight)
“How much money is there is all?”
(Thirty-eight cents) |

2. Show 5 more pennies (fig. 7.7). “How many pennies are here?” (Five)

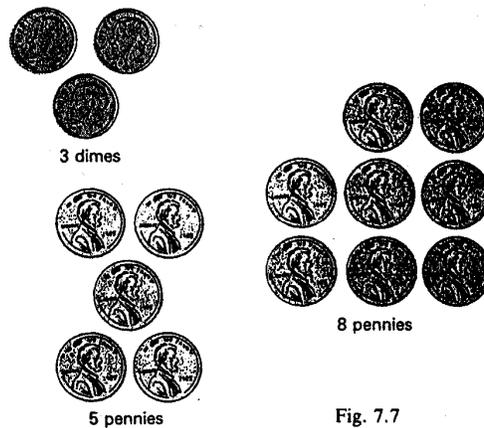


Fig. 7.7

3. Move 2 pennies from the group of 5 to the group of 8 pennies. “How many more pennies do you need to put with the eight to make ten?” (Two)
“What is thirty-eight plus two?” (Forty)
“How many pennies are left over from the group of five?” (Three)
“What is forty plus three?” (Forty-three)
“Yes, $38 + 5$ is 43. We can think 38 plus 2 more is 40, and 3 more is 43.”

Here are some other types of mental computation that can be developed in a similar manner.

1. Mentally counting by fives, tens, twenty-fives, fifties, and hundreds
2. Adding and subtracting multiples of 100
3. Adding and subtracting single-digit numbers and two-digit or three-digit numbers with and without renaming
4. Multiplying single-digit numbers by multiples of 10 and 100
5. Doubling a number and finding half of a number for multiples of 10 and 100

SUMMARY

Children in the primary grades are capable of learning many estimation skills. However, they need instruction to do so. Also, children’s mental computation skills need to be developed if they are to become proficient at estimating. The activities described here are a sampling of some that are appropriate for young children. Children with experiences like these in the primary grades will not only become better at estimating and mental computing but also develop a sense of number. This number sense goes far beyond the ability to compute with paper and pencil. It forms an excellent foundation on which problem-solving and logical-reasoning skills can be based.