

Task

8

Overview

Find and compare areas.
Use visual and/or
numerical reasoning.

Grass for Goats

Short Task

Task Description

Students are given the context of a goat grazing in a yard. Students determine the area of grass that the goat can reach when it is kept on a chain that is 3 meters long.

Assumed Mathematical Background

It is assumed that students have had experience finding the areas of circular regions.

Core Elements of Performance

- use visual, geometric, and/or numerical reasoning to find circular regions
- compute and compare areas

Circumstances

Grouping: Students complete an individual written response.

Materials: compass and ruler

Estimated time: 20 minutes

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Grass for Goats

This problem gives you the chance to

- *use visual, geometric, and numerical reasoning*
- *calculate and compare areas*

The Jacobsens keep their goat on a chain that is 3 meters long.

1. If they chain the goat to a metal hook in the center of their yard, what is the area of the grass that the goat can reach to eat? Sketch a picture and show your work.

2. Sometimes the Jacobsens chain the goat to the corner of a shed that is 5 meters by 4 meters. The 3-meter chain is

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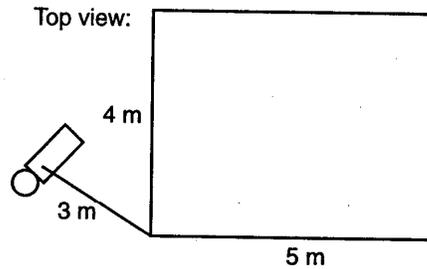
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attached to the base of the wall at ground level. What is the area of grass that the goat can reach? Show and explain your work.



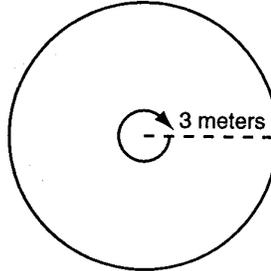
3. Suppose the goat was chained at ground level to the center of the 4-meter shed wall. Would the amount of grass the goat can reach be greater than what he could reach when chained to the corner of the shed? Justify your answer.

A Sample Solution

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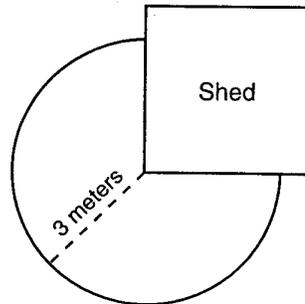
1. The goat can reach 3 meters in any direction. Therefore, the goat can reach a circular area of radius 3 meters.

$$\text{Area} = \pi \times 3^2 = 9\pi \approx 28.27 \text{ square meters}$$



2. The goat can still reach 3 meters in any direction except for the area covered by the corner of the shed. The corner of the shed makes a 90-degree angle so the shed covers the area of one-fourth of the original circle. Therefore, the goat can reach an area three-fourths of a circle of radius 3 meters.

$$\text{Area} = \frac{3}{4} (\pi \times 3^2) \approx 21.21 \text{ square meters}$$



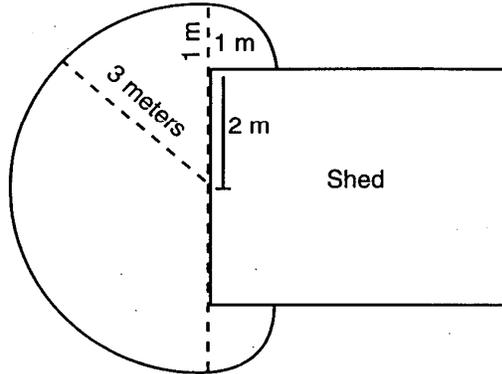
3. This question asks the student to compare two areas. It can be answered either by computing the second area and comparing numerical values or by making visual/geometric comparisons.

The goat can reach a semicircular area in front of the shed and he can reach around the shed on both sides as shown in the following picture.

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A computational response

The semicircular region is half of a circle of radius 3 meters: $\text{Area} = \frac{1}{2} (\pi \times 3^2) \approx 14.14$ square meters. Since the shed's corners are right angles, the two additional regions on the sides of the shed are each a quarter of a circle of radius 1 meter. Taken together, they are one half of a circle of radius 1 meter: $\text{Area} = \frac{1}{2} (\pi \times 1^2) \approx 1.57$ square meters. Add these two areas to get the total area, which is approximately 15.71 square meters.

The amount of grass the goat can reach when chained to the center of the shed wall is less than when chained to the corner since 15.71 square meters is less than 21.21 square meters.

A visual response

When the goat is chained to the corner of the shed, he can reach the area in Figure 1 below. When he is chained to the center of the shed, he can reach the area in Figure 2 below. The semicircles A and C have the same area. If you superimpose regions D and E on B, you can see in Figure 3 that they do not cover B. So the area the goat reaches in Figure 2 (chained to middle of the shed) is less than in Figure 1 (chained to the corner).

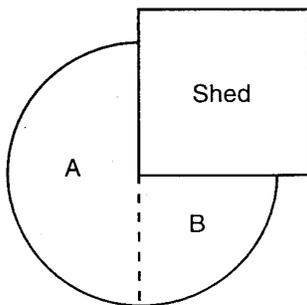


Figure 1

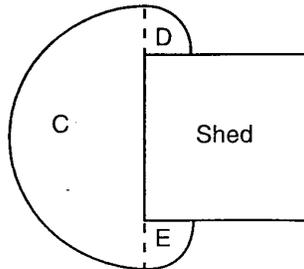


Figure 2

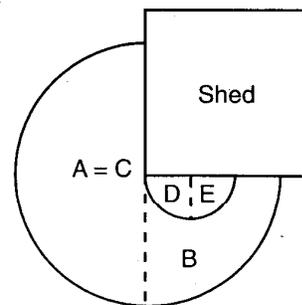


Figure 3